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PART II.

MENTAL ARITHMETIC

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W. J. Gage & Co's Mathematical Series.

MENTAL ARITHMETIC,

—BY—

J. A. McLELLAN, M.A., LL.D.,

Inspector of High Schools, Ontario.

PART II.

PERCENTAGE AND ITS APPLICATIONS,
VARIOUS RULES,
GENERAL ANALYSIS.

Authorized for use in the Schools of Manitoba.
Authorized for use in the Schools of British Columbia.
Authorized for use of Model School Students' of Ontario.

EIGHTH EDITION.

PRICE 45 CENTS.

W. J. GAGE & COMPANY,
TORONTO.

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PREFACE.

THE SECOND PART of the Canadian Mental Arithmetic is now submitted to the public.

It contains a great variety of properly classified questions in Percentage and its applications, Stocks and Shares, Interest, Discount, &c., together with solutions of almost every type of question likely to be met with in any treatise on arithmetic. Many of these solutions are original,† and are now published for the first time. It is believed that students who receive a fair training in the methods of the following pages, will find no difficulty with any treatise on written arithmetic; and that systematic mental drill will produce better arithmeticians, and at a less expenditure of *time* and energy than the rule and routine methods that have too long prevailed.

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† The elegant solution (No. 1), page 17, was suggested *independently* by Mr. Fessenden, of Brampton, an accomplished mathematician. It is substantially the same as that given in the *preface* to the "EXAMINATION PAPERS IN ARITHMETIC."

The publishers desire to express their great satisfaction with the very favorable reception accorded to the "FIRST PART" by the most experienced teachers in the Dominion, and venture to hope that the "Second Part" will meet with an equally kind reception, and prove a still more valuable aid to both teachers and students.

TORONTO, 1st Nov., 1878.

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CHAPTER I.

PERCENTAGE AND ITS APPLICATIONS.

Section I.—Introductory.

EXAMPLES.—1.

1. A boy spends 1 cent of every 5 cents he has: how many does he spend of 20 cents?

> *Sol.*—He spends 1 of every 5; there are 4 times 5 in 20 $\therefore$ he spends 4 times 1, or 4 $(=20\times1\div5$, or $=20\times\frac{1}{5})$.

2. A boy spends 2 cents of every 6 he has: how many does he spend of 30?

> *Sol.*—He spends 2 out of every 6; there are 5 sixes in 30 $\therefore$ he spends 5 times 2 or 10 $(30\times2\div6=30\times\frac{2}{6})$.

3. If he spends 4 out of every 10, how many does he spend of 60?

> *Sol.*—He spends 4 of every 10; there are 6 tens in 60 $\therefore$ he spends 6 times 4, or 24 $(=60\times4\div10=$ also $60\times\frac{4}{10}=60\times.4)$.

4. A boy gave away 6 marbles of every 20 he had: how many did he give away of 40? of 60? of 100?

5. A man expended in groceries \$6 of every \$10 he had: how much did he spend of \$45?

> *Sol.*—He spent \$6 of every \$10; there are $4\frac{1}{2}$ tens in 45 $\therefore$ he spent $4\frac{1}{2}$ times \$6=\$27 $(=45\times6\div10$, also $=45\times\frac{6}{10})$.

6. A man had 300 sheep, he sold 8 out of every 100: how many did he sell?

Sol.—He sold 8 of every hundred; there are 3 hundred ∴ he sold 3 times 8, or 24 ($=300\times8\div100$, or $=300\times\frac{8}{100}$).

7. A man paid away $7 out of every hundred he had: how much did he pay out of 200? 400? 500? 800? 900?

8. A man received $8 for every $100 lent for a certain time: how much did he receive for $350?

Sol.—He received $8 for 1 hundred; there were 3½ hundred lent ∴ he received $3\frac{1}{2}\times8$ $=\$28$ ($=350\times8\div100=350\times\frac{8}{100}$).

9. A man borrowed money for a time and paid $8 for every hundred: how much did he pay on $550? $525? $575?

Instead of using the phrases 2 *on every hundred*, 3 *on every hundred*, 4 *on every hundred*, 5 *on every hundred*, 6 *on every hundred*, &c., &c., we say 2 *r cent*. 3 *per cent*. 4 *per cent*. 5 *per cent*. &c., &c., the words *per cent*. meaning "on every hundred."—*Note.*—The symbol % is often used for the words *per cent*.

10. How much is 5 per cent of $300? $400? $800?

11. How much is 8 per cent. of $325? $450? $575?

Sol.—In 325 there are 3¼ hundreds ∴ 8 per cent. $=3\frac{1}{4}\times8=26.$ $=(325\div100)\times8=$ $325\times\frac{8}{100}$, also $=325\times.08$.

12. 5 is what part of 100? 6 is what part of 100? 7? 8?

13. What part of a hundred is 5 per cent. of it? 6 per cent.? 7 per cent.? 8 per cent?

14. Since 5 per cent. of $100=\frac{1}{20}$ of it, how much will 5 per cent of 200 be? 300? 400? 700? 900?

Sol.—Since 5 per cent. of $100=\frac{1}{20}$ of 100, 5 per cent of $200=\frac{2}{20}$ of $100=\frac{1}{20}$ of 200. So 5 per cent. of $300=\frac{3}{20}$ of $100=\frac{1}{20}$ of 300, &c.

15. What part of 545 is 5 per cent. of it?

Sol.—5 per cent. of $\$545=\$\frac{545}{100}\times5=\$545\times\frac{5}{100}=\$545\times\frac{1}{20}$ also $=\$545\times{\cdot}05$.

Or 100 per cent. of $\$545=\$545$.
.·. 1 per cent. " $=\$\frac{545}{100}$.
And 5 per cent. " $=\$\frac{545}{100}\times5=\$545\times\frac{1}{20}$.

16. What part of any number is 5 per cent. of it?

Sol.—100 per cent. of the number=the number. 1 per cent of the number$=\frac{1}{100}$ of the number .·. 5 per cent. of the number$=\frac{5}{100}=\frac{1}{20}$ of the number.

17. What is 5 per cent. of 80? of 60? of 40? of 85? of 90?

Sol.—5 per cent. of $80=80\times\frac{1}{20}=4$; or$=(80\div100)\times5=\frac{4}{5}\times5=4$; or$={\cdot}80\times5=4$; or $80\times\frac{5}{100}=80\times{\cdot}05=4$.

18. What is 8 per cent. of \$325?

Sol.—8 per cent. of $\$325=325\times\frac{8}{100}$ (or $\frac{2}{25}$) $=28$; or$=\frac{325}{100}\times8=3\frac{1}{4}\times8=28$; or$=3{\cdot}25\times8=28$; or$=325\times{\cdot}08=28$.

19. What then are the various ways of finding *any per cent.* of a number?

EXAMPLES.—2.

1. What part of a number is 6 per cent. of it? 3 per cent. of it? 4 per cent. of it? 8 per cent. of it?

2. What part of a number is 5 per cent. of it? 10 per cent.? 15 per cent.? 22 per cent.? 40 per cent.? 55 per cent.? 12 per cent.? 80 per cent.? 35 per cent.? 65 per cent.? 135 per cent.?

3. What part of a number is $12\frac{1}{2}$ per cent. of it? $18\frac{3}{4}$ per cent.? $6\frac{1}{4}$ per cent.? $8\frac{1}{4}$ per cent.? $37\frac{1}{2}$ per cent.? $2\frac{6}{7}$ per cent.? $10\frac{5}{12}$ per cent.? $87\frac{1}{2}$ per cent.?

4. What part of a number is $\frac{1}{2}$ per cent. of it? $\frac{3}{4}$ per cent.? $\frac{5}{8}$ per cent.? $\frac{7}{10}$ per cent.? $\frac{5}{12}$ per cent.? $\frac{7}{8}$ per cent.?

5. What per cent. of a number is $\frac{1}{5}$ of it?

Sol.—The entire number is 100 per cent. $\therefore$ $\frac{1}{5}$ of it $=20$ per cent.

6. What per cent. of a number is $\frac{1}{3}$ of it? $\frac{2}{6}$ of it? $\frac{3}{4}$ of it? $\frac{6}{25}$ of it: $\frac{2}{3}$ of $\frac{3}{4}$ of it?

7. What per cent. of 16 is 4?

Sol.—4 is $\frac{1}{4}$ of 16, and $\frac{1}{4}$ of a number is 25 per cent. of it.

8. What per cent. of 140 is 5? 10? 14? 21? 35? 105? 112?

9. A man bought a horse for \$145, and paid 40 per cent. of it in cash and the balance by note: find the amount of the note.

Sol.—40 per cent. of $\$145=\frac{2}{5}$ of $145=\$58$ $\therefore$ note $=\$145-\$58=\$87$.

10. A boy lost 20 per cent. of his marbles, sold 25 per cent. of the remainder, and had 36 left: how many had he at first?

Sol.—20 per cent lost$=\frac{1}{5}$ loss .·. $\frac{4}{5}$ rem.: 25 per cent. of this sold$=\frac{1}{4}$ sold .·. $\frac{3}{5}$ rem. $=36$, .·. &c.

11. 80 per cent of $250 is $62\frac{1}{2}$ per cent. of what my watch cost: find the cost.

12. In a school, 20 per cent. of the scholars are in the 5th class, each of the next three classes contains $18\frac{1}{3}$ per cent. of the remainder: what percentage of the school is in the primary class?

Sol.—20 per cent. in 5th class ∴ 80 per cent. $=\frac{4}{5}$ in the other classes; $18\frac{1}{3}$ per cent. $\times 3=$ 55 per cent. $=\frac{11}{20}$, and $\frac{11}{20}$ of $\frac{4}{5}=\frac{11}{25}$ in three of remaining classes ∴ $\frac{4}{5}-\frac{11}{25}=\frac{9}{25}=36$ per cent. in last class.

Examples.—3.

1. If I add $\frac{1}{2}$ of unity to itself, what fractional part of the sum must I take off so that the remainder may be unity?

2. What part taken from a number is equal to $\frac{1}{4}$ of the remainder?

Sol.—The remainder $+\frac{1}{4}$ of rem. $(=\frac{5}{4}$ rem.$)=$ given number .·. $\frac{4}{5}$ of number $=$ rem., and $\frac{1}{5}$ has to be subtracted.

3. To a number $\frac{1}{8}$ of itself is added, what part must be taken from the sum to get the number?

4. What part taken from a number is equal to $\frac{2}{5}$ of the remainder?

5. To a certain number $\frac{3}{8}$ of itself is added: what part must be subtracted from the sum to get the number?

6. How does the numerator of the part subtracted, compare with the numerator of what is added?

7. How does the denominator of what is taken off compare with the numerator and the denominator of what is added?

8. What per cent. taken from a number is equal to 40 per cent. of the remainder?

> *Sol.*—Rem. + 40 per cent. of rem. = 140 per cent. of rem. = $\frac{7}{5}$ of rem. = given number ∴ rem. = $\frac{5}{7}$ of given number, and $\frac{2}{7}$ or $28\frac{4}{7}$ per cent. must have been taken off.

9. What per cent. subtracted from a number is equal to 25 per cent. of the remainder?

10. 30 per cent. is added to a number: what per cent. must be taken from the sum to give the original number?

11. When $12\frac{1}{2}$ per cent. is added, what per cent. subtracted will give the number that was increased?

12. 40 is 8 per cent. of what?

> *Sol.*—8 per cent. = $\frac{8}{100}$ of it = 40 ∴ $\frac{1}{100}$ = 5, and number = 500.

13. If I take off $\frac{1}{5}$ of my price when selling an article, what per cent. of my price is left?

14. If I ask 96 cts. for a book and take off $12\frac{1}{2}$ per cent., how much do I get for it?

15. What per cent. must I add to the price of an article, so that when I take off 10 per cent. for a customer I may neither lose nor gain?

Section II.—Commission.

EXAMPLES.—4.

1. A man received $45 for selling goods on a commission of $2\frac{1}{2}$ %: find the price at which the goods sold.

Sol.—$2\frac{1}{2}\% = \frac{1}{40}$ of price $= \$45 \therefore$ price $= 40 \times 45 = \$1,800$.

2. Paid $3\frac{1}{8}$ % for collecting a debt of $320: what sum do I receive?

3. An auctioneer sells goods on a commission of $2\frac{1}{4}$ %: what will he receive on a sale of 2,500 dollars' worth?

4. A broker sold $6,000 Bank of Commerce stock, on a commission of $\frac{5}{8}$ per cent.: find amount of his commission.

5. At $\frac{4}{5}$ per cent. what will it cost to buy a bill of exchange for $1,250?

Sol.—$\$1,250 = 12\frac{1}{2}$ hundreds $\therefore$ com. $= \frac{4}{5} \times 12\frac{1}{2} = \$10$.

6. A Toronto merchant consigns 12,000 bushels of wheat to his agent in Montreal, who sells it at $1.20 a bushel, on a commission of $1\frac{3}{4}$ per cent: find the amount of the commission.

7. A planter paid his agent in New York $125 for selling 40,000 lbs. of cotton, on a commission of $2\frac{1}{2}$ per cent.: for what price per lb. was the cotton sold?

Sol.—$\frac{1}{40}$ of cotton $= 1,000$ lbs. went for commission $=$ in value $\$125 \therefore$ cotton $\$\frac{1}{8}$, or $12\frac{1}{2}$ cts.

8*a*. An agent receives \$225 for selling goods to the value of \$5,000: find his rate per cent. of commission.

> *Sol.*—\$5,000 gives \$225 com.: \$1 gives $\frac{225}{5000}$ $= \frac{9}{200}$, and \$100 gives $\frac{9}{200} \times 100 = \frac{9}{2} = 4\frac{1}{2}$. Or 50×100 (=5,000), gives 225 ∴ 100 gives $225 \div 50 = 4\frac{1}{2}$.

8. An agent received \$117 for selling 800 barrels of flour, at \$6.50: find his rate of commission.

9. A tax collector receives $1\frac{1}{2}$ per cent. for all sums paid within 30 days, and 3 per cent. for all paid after that time: the taxes are \$45,000, of which \$25,000 is paid within 30 days: find the whole amount of the collector's fees.

10. A druggist sends his agent \$630 with which to buy goods, after deducting his commission, 5 per cent. on the price paid for the goods: find the agent's commission.

> *Sol.*—For every \$105 sent, the agent receives \$5 ∴ he receives $\frac{1}{21}$ of whole amount sent= $\frac{1}{21}$ of \$630=\$30.

11. A manufacturer sent \$2,601 to a wool dealer with which to purchase wool, allowing him 2 per cent. for the money paid out: how much would be invested in wool?

12*a*. An agent *sold* flour on a commission of 3 per cent., and with the proceeds, *minus* his commission for both transactions, purchased tea on a commission of 2 per cent. on the price paid for it; his entire commission was \$200: find amount received for the tea.

Sol. (1.)—3 per cent. + 2 per cent. = 5 per cent. If 5 per cent. (= $\frac{1}{20}$) had been charged on the whole amount of sales, the com. would have been 2 per cent. of $200 = $4 more, *i.e.*, the entire com. would have been $200 + $4 = $204 = $\frac{1}{20}$ of sales, which ∴ amounted to 20 × 204 = $4080. Further: if the 5 per cent. com. had been taken on amount of *purchase* money, the entire com. would have been 3 per cent. of $200 = $6 less than it was, = 200 — 6 = $194 = $\frac{1}{20}$ of amount of *purchase*, which = ∴ $3880. See "Exam. Papers in Arithmetic."—*Preface.*

Sol. (2.)—It will be found that on every $102 from *sale* there is $5 entire commission: Suppose we allow for com. on *purchase*, $2 of the 102, leaving $100. For com. on *sale*, $3 of the $100, leaving $97=$5 com.

In the former case we have charged 2 per cent. of $3=6 cents *too much.* But in the latter case we have charged 3 per cent. of $2 =6 cents *too little.* *i.e.*, the *excess* equals the deficit, and we have still $5 entire commission. Then, $\frac{5}{102}$ of *sales* = $200, $\frac{1}{102}$ = $40, and the whole=$4,080.

It is seen from the above that the com.= $\frac{5}{102}$ (= $\frac{5}{100+2}$) of the *sales*; and equals $\frac{5}{97}$ (= $\frac{5}{100-3}$) of *purchase* money.

Sol. (3.)—The formal analysis is: Take $100 for reference. $3 = com. on *sale*: $\frac{2}{102}$ of (100 —3) = com. on *purchase*. ∴ Entire com. = $3 $-\frac{6}{102}+\frac{200}{102}=\frac{600}{102}=\frac{5}{102}$ of 100.

NOTE.—If com. on *sale* is 4 per cent. and on *purchase* 3 per cent., the entire com. $=\frac{4+3}{103}=\frac{7}{103}$ of *sale* money, and $=\frac{4+3}{98}$; (*i.e.*, $\frac{7}{100-4}$ of purchase money). And generally if we have m per cent. on *sales*, and n per cent. on purchase, the entire com. $=\frac{m+n}{100+n}$ of *sale* money, and $=\frac{m+n}{100-m}$ of *purchase* money.

12*b*. A merchant sold tea on a commission of 5 per cent., and purchased (as in last Ex.) cotton on a com. of 3 per cent.; his entire com. was $400: find amount received for the goods.

12*c*. In the last question, find also the amount invested in cotton.

12. A merchant sends his agent $390 to expend for goods, after deducting his commission at 4 per cent. on the price paid for the goods: find the agent's commission.

13. A merchant sends his agent $5,200 with which to purchase wheat, allowing him a commission of 4 per cent. on the price paid; he paid $1.25 a bushel: how many bushels did he buy?

14. A commission merchant received 5 per cent. for selling goods, and purchased with the proceeds (after deducting his commission for both transactions) a quantity of oats on a commission of 5 per cent. on the amount paid for the oats: he paid 57 cents a bushel, and his entire commission was $60: how many bushels did he buy?

Section III. — Insurance.

EXAMPLES.—5.

1. A man effected an insurance of $1,575 on his house, at 2 per cent. premium : how much did he pay?

2. What will it cost to insure $2,400 worth of furniture at $\frac{3}{8}$ per cent. premium?

3. A produce dealer paid $25 for an insurance on goods valued at $800 : what rate of premium did he pay?

4. An insurance company which charged a rate of $\frac{5}{8}$ per cent. received $22.50 for insuring a house to the amount of 75 per cent. of its value : find its value.

Sol.—\$$\frac{5}{8}$ from 100 ∴ \$1 from \$160 and \$$22\frac{1}{2}$ from $22\frac{1}{2} \times 160 =$ \$3,600 ; *or*, $\frac{5}{8}$ from \$100 ∴ $22\frac{1}{2} \div \frac{5}{8} = 36 =$ no. hundreds, &c.

5. An insurance agent took a risk on a ship at $2\frac{1}{2}$ per cent., and immediately re-insured 80 per cent. of the risk in another company at 2 per cent. : the premium received exceeded the premium paid by $225 : for how much was the ship insured?

6. A farmer paid an agent $8.25 for insuring his barn and its contents at $\frac{1}{2}$ per cent. : for what amount did he insure?

Sol.—\$$\frac{1}{2}$ prem. on \$100 insured, and \$1 on \$200 ∴ \$$8\frac{1}{4}$ on \$$200 \times 8\frac{1}{4} =$ \$1,650.

7. For how much must I insure at 4 per cent. a property worth $3,600, so as to suffer no loss in case the property is destroyed?

> *Sol.*—I must receive both value of property and premium paid, and since the premium is 4% of amount insured, the property must be 100% —4% = 96% = $\frac{24}{25}$ of amt. insured, ∴ $\frac{24}{25}$ of this amt. = \$3,600 and amt. = \$3,750.

8. For how much must a house costing \$1,470 be insured at 2 per cent., so that the owner, in case of loss, may recover its value and the premium paid?

9. A ship's cargo is valued at \$28,500: what premium must be paid at 5 per cent. so that in case of shipwreck the insurer may suffer no loss?

10. My property is worth \$4,800: what is the tax on it at $1\frac{1}{4}$ per cent.?

10*a*. The tax on a lot worth \$500 is \$6: what is the rate on the dollar?

> *Sol.*—\$500 gives \$6. ∴ \$1 gives \$$\frac{6}{500}$ = $1\frac{1}{5}$ cents = 12 mills.

11. At 17 mills on the dollar my income tax is \$25.50: on what amount do I pay income tax?

12 A man who owns property to the amount of \$4,250 pays \$72.25 in taxes: find the rate in the dollar.

13. A tax collector pays \$11,400 to the treasurer of a municipality, after deducting his commission at 5 per cent.: how much did he collect?

> *Sol.*—5% = $\frac{1}{20}$ for collecting ∴ $\frac{19}{20}$ left = \$11,400 ∴ $\frac{1}{20}$ = \$600, etc.

14. For how much must a town assess itself so as to allow the collector 4 per cent. and still realize \$18,000?

15. What is the duty on 750 gallons of wine at $1.20 a gallon?

16. A lot of books is invoiced at $775: find the duty at 5 per cent.

17. Paid a specific duty of 50 cents a gallon and an *ad valorem* duty of 17½ per cent. on 500 gallons of brandy: find the amount of duty paid.

18. A man owns $\frac{7}{64}$ of a ship worth $40,000. It is insured for a certain voyage at 2 per cent: find amount of his premium.

Section IV.—Profit and Loss.

EXAMPLES.—6.

1. A man bought tea at 45 cts. a lb., and sold it at 55 cts.: find his gain per cent.

Sol.—45 gains 10 $\therefore$ 1 gains $\frac{10}{45}$, and 100 gains $\frac{10}{45} \times 100 = 22\frac{2}{9}$.

2. A merchant bought flour for $6 a bbl., and sold it at $7.50: find his gain per cent.

3. Bought cloth at 80 cts. a yard, and sold it at $1.10: find the gain per cent.

4. Bought a quantity of wheat at $1.20 per bushel, and sold it at $1.10: find the loss per cent.

5. An apple-woman bought apples at $1.25 per hundred, and sold them at 2 cents each: what was her gain per cent?

6. Invested $4,500 in flour at $6.75 a bbl.; sold it at $7.50 a bbl.: find entire gain.

7. A man sold a horse for $150, which was $\frac{3}{4}$ of what he paid for it: find his loss per cent.

8. A merchant bought 300 barrels of flour at $5 a bbl., sold two-thirds of it at $8 a bbl., and the rest at cost: find his gain per cent.

9. A grocer sells 8 lbs. of tea for what 9 lbs. cost him: what is his gain per cent?

Sol.—He gains 1 lb. on 8, or $\frac{1}{8}$ on 1 $\therefore$ gain $= 12\frac{1}{2}\%$.

10. Bought a horse for twenty per cent. less than $150, and sold him for 10 per cent. more than $150: find gain per cent.

11. I sold goods to A at 10 per cent. profit; A sells them to B at 20 per cent. profit: what per cent. should I have made if I had sold directly to B at the price A received?

12. A merchant marks his goods at an advance of 25 per cent. on cost; what per cent. must he reduce his marked price to sell certain damaged goods at cost?

Sol.—$\frac{5}{4}$ of cost $=$ selling price $\therefore$ $\frac{4}{5}$ of latter $=$ cost, $\frac{1}{5}$ off selling price $=$ 20 per cent. off.

EXAMPLES.—7.

1. Bought tea at 50 cents per lb., and sold it so as to gain 25 per cent.: find the selling price.

Sol.—25 per cent. $= \frac{1}{4}$, and $\frac{1}{4}$ of $50 = 12\frac{1}{2}$ $\therefore$ $50 + 12\frac{1}{2} = 62\frac{1}{2}$. *Ans.*

2. Land was bought at $20 an acre, and sold so as to gain 45 per cent.: find the selling price.

3. A grain merchant bought wheat at $1.20, and sold it at a loss of 15 per cent.: find the selling price.

4. A merchant "marks down" a lot of old goods $12\frac{1}{2}$ per cent.: what will be the price of cloth for which his price had been $3.68 a yard?

5. A town had a population of 4,045 in 1870; in 1875 the population had decreased $11\frac{1}{9}$ per cent.: find its population at latter date.

6. By selling cotton at 12 cents a yard, there is a gain of 20 per cent.: what was the cost price?

> *Sol.*—20 per cent. $= \frac{1}{5} \therefore 1\frac{1}{5}$ cost $= 12$, and cost $= 10$.

7. A merchant sold tea at 64 cents, gaining $33\frac{1}{3}$ per cent.: find the cost of the tea.

8. A man sold his house for $2,400, which was 2 per cent. below cost: find the cost.

9. A drover sold 120 sheep for $504, gaining 2 per cent.: what did they cost him a piece?

10. A grocer bought coffee so that he could sell i at 36 cents and make a profit of $33\frac{1}{3}$ per cent.: fin the cost.

11. By selling cigars at 65 cents a dozen, 30 pe cent. was gained: what selling price would hav gained 60 per cent.?

> *Sol.*—$\frac{13}{10}$ cost $= 65 \therefore$ cost $= 50$, add 60 per cent., we have $50 + 30 = 80$.

12. B sells a lot to C, and gains $12\frac{1}{2}$ per cent. C sells it to D for $1,180, gaining 18 per cent.: wha did the lot cost B?

13. A merchant gains 30 per cent. by selling muslin at 39 cents a yard: by what selling price would he lose 40 per cent.?

> *Sol.*—$\frac{13}{10}$ cost $= 39 \therefore$ cost $= 30 - 40$ per cent. of this $= 18$c.

EXAMPLES.—8.

1. A horse was sold for $180, which was 10 per cent. less than it cost: what was the cost?

2. By selling coffee at 30 cents a pound, there is a loss of 20 per cent.: what selling price would have gained 20 per cent.?

> *Sol.*—Sold for $\frac{4}{5}$ of cost; to gain 20 per cent. it must sell for $\frac{6}{5}$ of cost, which is one-half more than $\frac{4}{5}$ $\therefore$ 45 cents.

3. A man sold a horse and carriage for $480, losing 4 per cent.; if he had sold them at $600 what would he have gained per cent.?

4. A man sold two lots at $400 each: on one he lost 20 per cent., and on the other he gained 20 per cent: how much did he gain or lose on the whole?

> *Sol.*—$400=$1\frac{1}{5}$ cost of one=$\frac{4}{5}$ cost of the other $\therefore$ costs are $500, 333$\frac{1}{3}$, etc.

5. What must be the marked price of cloth that cost $150 a yard, so that the seller may reduce his price 10 per cent. and still make 20 per cent. profit?

> *Sol.*—$\frac{9}{10}$ of *marked* price=$1\frac{1}{5}$ cost=$1.80 $\therefore$ marked price=$2.

6. If 10 per cent. more is gained by selling a horse for $280, than by selling him for $260, find his cost price.

7. A merchant invested $1,096 in broadcloth; he marked it at an advance of 20 per cent. on cost, but finally took 20 per cent. less than he asked: find his entire loss

Sol.—100 sold for 120: 20 per cent. of this$=$ 24, $120-24=96$ ∴ 4 lost on $100=4$ per cent.

8. By selling a house which cost \$4,800, I gained $7\frac{9}{13}$ per cent. of the selling price: find the selling price.

9. By selling tea which cost 48 cents a lb., I lost $11\frac{1}{9}$ per cent. of the selling price: what was the loss per cent. on the cost?

Sol.—$\frac{1}{9}$ of selling price lost ∴ $\frac{10}{9}$ of do.$=$cost ∴ $\frac{1}{10}$ cost lost$=$ $10\frac{1}{2}$ per cent.

10. I sold one farm which cost \$6,000, at a gain of $16\frac{2}{3}$ per cent. of the selling price, and another which cost \$7,200 at a loss of 20 per cent. of the selling price: find my whole gain or loss per cent.

11. How many yards of cloth at \$4 a yard must a merchant buy, so that by selling it at 10 per cent. profit he may gain \$24?

Sol.—10 per cent. of $\$4=\frac{2}{5}=$gain on 1 yard and $\$24=$gain on $24\div\frac{2}{5}=60$ yards.

12. At what must I mark goods that cost 60 cents per yard, so that I can take off 20 per cent. for a customer and neither gain nor lose?

Sol.—$\frac{1}{5}$ off marked price$=$cost ∴ $\frac{1}{4}$ (or 25 per cent.) added to cost$=$selling price.

13. At what must I mark goods that cost 50 cents per yard, so that I can take off 10 per cent. for a customer and have 13 cents profit?

14. At what must I mark a book which cost 97 cents, so that after taking off $12\frac{1}{2}$ per cent. for a customer I may lose 6 cents.

15. Cloth which cost 96 cents a yard was sold at a profit of 25 per cent., and $16 was gained in all : how many yards were there ?

16. A boy bought peaches at the rate of 3 for 10 cents. and sold them at the rate of 5 for 20 cents.: what did he gain per cent. ?

17. At what must I mark goods worth 96 cents per yard, so that I can take of 20 per cent. for a customer and have 25 per cent. profit ?

> *Sol.*—$96+25$ per cent. of $96=120$; $\frac{1}{5}$ off marked price$=120 \therefore 120+\frac{1}{4}$ of $120=150$.

18. At what must I have marked goods that cost $1.80 per yard, that after taking off 10 per cent. I lost 5 per cent. ?

19. Sold a lot for $230, which was 8 per cent. less than it cost : had it been sold for $300 what would have been the gain ?

20. Bought a quantity of apples at 80 per cent. off the market price, and sold them for 10 per cent. more than the market price : find the gain per cent.

21. A man sells wheat at 90 cents a bushel, and gains $12\frac{1}{2}$ per cent. : what will he gain or lose per cent. by selling it at 85 cents a bushel ?

22. A grocer bought 40 lbs. of tea for 80 cents a lb., and intended to sell it so as to gain 25 per cent., but by unintentionally using a false balance, he really gains $33\frac{1}{3}$ per cent. : how many ounces did he give for a lb ?

> *Sol.*—Cost $=$ \$32, selling price $=$ \$32 $+\ \frac{1}{3}$ of \$32 $=$ \$$42\frac{2}{3}$, which at $1 nominal lb.

represents $42\frac{2}{3}$ lbs. (false) $= 40$ (true) $\therefore 1$ false $= 40 \div 42\frac{2}{3} = \frac{15}{16}$ true .•. 15 ounces to lb.

23. I bought a horse for $100, which was $16\frac{2}{3}$ per cent. less than his real value, and sold him for 10 per cent. more than his real value on a credit of $3\frac{1}{5}$ years, money being worth 10 per cent. per annum: did I gain or lose, and how much?

24. The workmen in a New Brunswick ship-yard who work 11 hours a day, demand that the day's work be decreased by one hour; the employers agree to grant an equivalent to the demand by increasing the wages, the hours to remain the same: what increase per cent. in wages should be made?

25. The imports of Prince Edward Island amount to $1\frac{1}{2}$ million dollars annually, and the exports to 2 millions, each person on an average contributing $33\frac{1}{3}$ per cent. more to the exports than he receives from the imports, both for one person being $35: what is the population of the island?

26. A man marks his goods 20 per cent. above cost; 5 per cent. of his sales are bad debts on which he only receives 10 per cent., the cost of collecting is 2 per cent.: find his gain or loss per cent.

27. In a mixture of wine and water, 20 per cent. of the whole is water; 20 gallons more wine are added, and now the water is only 4 per cent. of the whole: how many gallons of wine were there at first?

Sol.—At first $\frac{1}{5}$ water $\frac{4}{5}$ wine, *i.e.*, wine = 4 times water. Then $\frac{1}{25}$ water $\frac{24}{25}$ wine, *i.e.*, wine = 24 times water .·. wine = 20 times water must have been added .·. 1 gallon of water and 4 wine.

28. A school-room not containing seats for six of the pupils, the number of seats was increased $33\frac{1}{3}$ per cent., and now there are seats for 12 more than the whole number of pupils, allowing two pupils to a seat: how many pupils were there in the school?

Section V.—Stocks and Shares.

Examples.—9.

1. How much cash will be realized by selling out $4,000 stock in Dominion 5's at $95\frac{1}{4}$?

Sol.—$100 brings $95\frac{1}{4}$ cash .·. $4,000 brings $40 \times 95\frac{1}{4}$ = $3,810.

2. What cash will be obtained by the sale of $7,800 3 per cent. stock at 89?

3. What amount will be received by selling out $4,500 Dominion 6's at $106\frac{3}{8}$, allowing brokerage $\frac{1}{8}$?

Sol.—Every $100 brings $106\frac{3}{8}—\frac{1}{8} = 106\frac{2}{8}$, &c.

4. Find amount realized by the sale of $5,000 Bank of Commerce stock at $120\frac{5}{8}$, allowing $\frac{1}{8}$ for brokerage.

5. How much stock can be bought at 92 for $2,760?

Sol.—92 will buy $100 stock .·. given sum will buy $\frac{2760}{92} \times 100 = 30 \times 100$ = $3,000.

6. How much stock can be bought at $92\frac{1}{2}$ for $1,480?

7. How much stock will $4,200 buy in the 4 per cents at 75 ?

8. How much Dominion Bank Stock at 120 can be bought for $5,400 ?

9. How much will $2,199 buy in the 3 per cents at $91\frac{1}{2}$, allowing brokerage $\frac{1}{8}$?

10. What semi-annual income will be derived from investing $9,000 in Dominion Bank Stock at 120, and paying 4 per cent. half yearly dividends ?

> *Sol.*—120 will buy 100 stock, which brings $4 income, and $\frac{4}{120} = \frac{1}{15}$ ∴ income is $\frac{1}{15}$ of investment ; $\frac{1}{15}$ of $\$9,000 = \$600$.

11. What income will be derived from an investment of $2,940 in 4 per cent. stock at 98 ?

12. What semi-annual income will be derived from investing $3,195 in stock selling at $88\frac{3}{4}$, and paying 3 per cent. half-yearly ?

13. What income will be received from investing $3,360 in $5\frac{1}{2}$ per cent. stock at 96 ?

14. What income is got from investing $7,450 in 3 per cent. stock at $74\frac{3}{8}$, allowing brokerage $\frac{1}{8}$?

> *Sol.*—100 stock costs $74\frac{3}{8} + \frac{1}{8} = 74\frac{1}{2}$, and brings $3 income, and $7450 \div 74\frac{1}{2} = 100$ ∴ income $= 100 \times 3 = 300$.

EXAMPLES—10.

1. If I invest in 7 per cent. stock at 120, what rate of interest do I obtain ?

> *Sol.*—$120 buys 100 stock, which entitles me to $7 ∴ $1 brings $\frac{7}{120}$ income $= \frac{7}{120} \times 100$, per cent. $= 5\frac{5}{6}$.

2. Bought stock in the 3 per cents at 80: what rate of interest was realized?

3. Invested £8,750 in the $3\frac{1}{2}$ per cents at $87\frac{1}{2}$: find the rate per cent. obtained from the investment.

4. Invested in railway stock selling at 85 and paying 5 per cent. dividends: find rate of interest.

5. What per cent. is obtained by investing in $3\frac{1}{2}$ per cent. stock at 75?

6. What amount of bank stock at 120 must be sold to produce $1,800 cash?

> *Sol.*—$120 cash is got from $100 stock, and $1 from $\frac{100}{120} = \frac{5}{6}$ ∴ 1,800 from $\frac{5}{6} \times 1,800 =$ 1500.

7. When the English funds are at 75, what stock must be sold to realize £125?

8. What amount of 8 per cent. stock at $125 must be sold to produce $3,750 cash?

9. What amount must be sold of Dominion 5's at $92\frac{1}{2}$ to produce $666?

10. What amount must be sold out of the $17\frac{1}{2}$ per cents at 128 to produce $1,024?

11. What sum must be invested in 8 per cent. stock at 120 to give income of $200?

> *Sol.*—8 arises from 120 ∴ 1 from 15 & 200 from $200 \times 15 = 3,000$.

12. What sum must be invested in 8 per cent. stock at 120 to give an income of $320?

13. What sum must be invested in the 5 per cents at 90 to give an income of $375?

14. What sum must be invested in the $4\frac{1}{2}$ per cents at 67 to produce $279 income?

EXAMPLES.—11.

1. Which is the better investment, insurance stock at 140 and paying 12 per cent. dividends, or bank stock at 120 and paying 9 per cent. dividends?

Sol.—In first case, 140 brings 12 .·. 1 brings $\frac{12}{140} = \frac{3}{35}$.

In second case 120 brings 9 .·. 1 brings $\frac{9}{120} = \frac{3}{40}$, and the first fraction is greater than the second .·. 1st is better investment.

2. Which is the better investment, the 6 per cents at 98½, or the 5 per cents at 85, brokerage in each case being ½?

3. Which is the better investment, 5 per cent. stock at 95, or 6 per cent. at 106?

4. A man having $12,500 Dominion Bank stock paying 8 per cent., sells out at 120, and invests in Bank of Commerce stock at 125, and paying 8½ per cent.: find alteration in his income.

Sol.—Income from D. B.—8 × 125 = $1,000. The D. B. sells at $\frac{1}{5}$ advance, giving $15,000 125 in B. C. gives 8½ .·. 1,000 gives 68, and 15,000 gives 68 × 15 = 1,020 .·. 1,020—1000 = $20 increase.

5. How much stock at 140 can be bought by selling out $14,000 of a different stock which is at 120?

Sol.—$1 of latter stock will buy $\$\frac{120}{140} = \$\frac{6}{7}$ of former .·. given amt. will buy $\$\frac{6}{7} \times 14{,}000 = \$12{,}000$.

6. How much stock at 95 can be bought by selling out $3,800 of a different stock at 120?

7. How much stock at $99\frac{3}{4}$ can be bought with the proceeds of $4,200 of a different stock at 95 ?

8. When gold is at $102\frac{1}{2}$, what is the discount on greenbacks ?

9. I invested a certain sum in railway stock at 80 and paying 5 per cent. dividends, and an equal sum in bank stock at 120, and paying 8 per cent. ; the difference in the two incomes was $12 : find the amount invested in each kind of stock.

> *Sol.*—The first pays $\frac{1}{16}$, the second $\frac{1}{15}$ (of amt. invested) $\therefore \frac{1}{15}-\frac{1}{16}=\frac{1}{240}$ of amt. =$12 $\therefore$ amt. = $3,880.

10. A certain sum was invested in 4 per cent. stock at 80, and an equal sum in 5 per cent. stock at 95 ; the difference in incomes was $10 : how much was invested ?

11. A sum of money was invested in insurance stock selling at 144, and paying 12 per cent. dividends, and twice the amount was invested in bank stock selling at 120, and paying 8 per cent. dividends ; the income from the latter investment was $125 greater than that from the former : how much was invested in each stock ?

12. A person sells £1,250 stock of the 3 per cent. consols, when the funds are at 96, and invests the proceeds in railway stock at 75, paying an annual dividend of $2\frac{1}{2}$ per cent. : find the alteration in his income.

13. How much 3 per cent. stock must be bought at $88\frac{1}{2}$ in order that by selling out at $88\frac{3}{4}$, there may be a gain of $21 ?

14. A person sells $2,400 stock at 95, which pays 5 per cent. in order to invest in 6 per cent. stock ;

what price must he pay for the latter that his income may remain unaltered.

Sol.—Income per ann. in 1st case $= \frac{5}{95} = \frac{1}{19}$ = income per ann. in the 2nd case $= \frac{6}{6 \times 19} = \frac{6}{114}$ $\therefore$ 114 is the price.

15. A man invests \$19,450 in Bank of Montreal stock at 194, and \$19,850 in Bank of Toronto stock at 198, paying in each case brokerage ½ per cent. on stock purchased: the former pays 6½ per cent., the latter 6¼ per cent. half-yearly dividends: find the total income for the half year.

Section VI.—Interest.

Examples.—12.

1. At 8 per cent. for 4 years, what part of the principal is the interest?

Sol.—Interest for one year $= \frac{8}{100}$, and interest for 4 years $= 4 \times \frac{8}{100} = \frac{8}{25}$.

2. At 8 per cent. for 5 years, what part of the principal does the interest equal?

3. At 6 per cent. for 5 years 8 months, what part of the principal equals the interest?

4. At 8 per cent. for 2 years 9 months, to what part of the principal is the interest equal?

5. Find interest on \$60 for 6 years, at 5 per cent.? at 6 per cent.? at 7 per cent.? at 8 per cent.? at 7½ per cent.?

6. What is the interest on \$600 for 2¼ years at 4 per cent.? 5 per cent.? 6 per cent.? 7 per cent.? 8 per cent.? 8¼ per cent.?

7. What is the interest on \$480 for 3 years 9 months, at 8 per cent.? at 6 per cent.? at 5 per cent.? at 10 per cent.?

8. Find the interest on \$480 for 5 years 10 months, at 6 per cent? at 8 per cent.? at 10 per cent.?

9. What is the interest on \$300 for five years, at $3\frac{1}{3}$ per cent.? at 6 per cent.? at $6\frac{2}{3}$ per cent.? at $7\frac{1}{5}$ per cent.?

10. What is the interest on \$800 at 8 per cent. for 3 months? for 4 months? for 5 months? for 7 months? for 9 months? for $7\frac{1}{2}$ months? for 10 months?

11. What is the interest on \$24.50, at 8 per cent. for 12 months? for 9 months? 6 months? 15 months?

12. What is the interest at 8 per cent. on \$137.50 for 1 year? for 15 months? 18 months? 14 months? 10 months? 8 months? 7 months? 9 months?

13. What is the interest on \$146 for 45 days, at 5 per cent.?

Sol.—45 days $= \frac{45}{365}$ year $= \frac{9}{73}$, $\therefore$ interest $= \frac{9}{73} \times 146 \times \frac{5}{100} = \$\frac{9}{10} =$ 90 cents.

14. What is the interest on \$73 at 6 per cent. for 35 days? 40 days? 50 days? 65 days? 90 days? 95 days? 115 days?

15. Find the interest on \$109.50, at 8 per cent. for 40 days? 45 days? 90 days? 120 days?

16. What is the interest on \$182.50 for 90 days, at 6 per cent.? at $7\frac{1}{2}$ per cent.? at 8 per cent.? at 10 per cent.?

17. Find the interest on \$240 for 8 months at 6 per cent.

Sol.—At 6 per cent. interest on $1 = $\frac{1}{2}$ cent per month = 4 cts. for 8 months ∴ interest on $240 = 240 × 4 cts. = $9.60.

18. Interest on $225 at 6 per cent. for 4 months? 6 months? 7 months? 8 months? 14 months? 13 months?

19. Interest on $240 for 11 months at 6 per cent.? at 7 per cent.? at 8 per cent.? at $7\frac{1}{2}$ per cent.? at 5 per cent.?

20. Find the interest on $800 for 42 days at 6 per cent.?

Sol.—Reckoning a year 360 days, the interest on $1 is a mill ($\frac{1}{1000}$ of $1) every 6 days, ∴ interest on the above = $\frac{42}{6}\times 800$ (mills) = $5.60.

NOTE.—The interest as found by this rule is greater than the true interest by $\frac{1}{73}$ of itself; in the last example the interest found is too great by nearly 8 cents.

21. What is the *amount* of $75 for 5 years at 8 per cent.?

Sol.—Interest = $\frac{2}{5}$ of $75 = $30 ∴ *amount* = $75 + $30 = $105.

22. Find the amount of $120 for 4 years at 6 per cent.? at 7 per cent.? at 8 per cent.? at 10 per cent.?

23. Find the interest, for 7 years 6 months, on $600 at 6 per cent., at $7\frac{1}{2}$ per cent., at 8 per cent., at 10 per cent.

24. A and B wish to divide the amount of $800 for 8 years at 5 per cent., so that A's part shall be 6 times B's.: find each man's share.

25. A's money is $500, which is $\frac{1}{4}$ of B's: what interest will each receive on his money for 15 months at 8 per cent.?

26. A, B, and C have together, $1200, of which B has 3 times as much and A 4 times as much as C: find the amount of each man's money for $2\frac{1}{2}$ years at 8 per cent.

Examples.—13.

1. What principal will, in $2\frac{1}{2}$ years at 8 per cent., give $30 interest?

Sol.—Interest=8 per cent.$\times 2\frac{1}{2}=\frac{1}{5}$ of principal $\therefore$principal=30\times 5$=$150.

2. What principal, in 2 years at 8 per cent., will give $16 interest? $12? $24? $19.20?

3. What principal, in $5\frac{1}{2}$ years at 8 per cent., will give $11? $22? $16.50? $13.20? $49.50?

4. I receive $160 interest for 2 years 8 months on a sum of money lent at 6 per cent.: find the sum.

5. The interest on $\frac{2}{3}$ of A's money for $6\frac{1}{4}$ years at 8 per cent., is $125: how much money has he at interest?

6. What principal will give $80 interest, if lent for 1 year and 8 months, at 9 per cent.?

7. I pay $98 annual interest on money borrowed at 7 per cent.: find the amount borrowed.

8. The interest on the sum of B's money and A's, for $2\frac{1}{4}$ years at 8 per cent. is $360.72: A's money is $204 more than B's: find each man's money.

Sol.—Interest $2\frac{1}{4}$ years $\times$ 8=18 per cent. $=\frac{9}{50}$ of joint sum=$360.72 $\therefore\frac{1}{50}$=$40.08 and whole=$2,004. $\therefore$ shares are $900, $1104.

9. The amount due on a note which had been on interest for 3 years and 4 months at 6 per cent. is $720: find the face of the note.

10. A had a sum of money at interest for 15 months at 8 per cent. and B 4 times as much for 18 months, at 6 per cent.; the sum of their interests was $23 : how much had each at interest?

11. What principal will produce 30 cents interest in 25 days at 6 per cent.?

Sol.—$\frac{25}{365} = \frac{5}{73}$ year ∴ interest $= \frac{5}{73} \times \frac{3}{50}$ of principal = 30 cts. and principal = $73.

12. The interest on a note for 100 days at 8 per cent. is $2.40 : find the face of the note?

13. The interest on a sum of money at 10 per cent. for 1 year and 20 days, is $77 : find the sum.

14. The amount required to pay a note which has run 150 days at 8 per cent. is $730 : find face of the note.

Examples.—14.

1. The interest on $400 at 5 per cent. for a certain time is $60 : find the time?

Sol.—At 5 per cent. $20 is the interest for one year ∴ $60 is the interest for $\frac{60}{20}$ years = 3 years.

2. In what time will $40 make $8 interest at 2 per cent?

3. I lent $720 at 6 per cent. and received $140 interest : for how long was the money lent?

4. The interest on $62.50 at 8 per cent. was $12 ; how long was it on interest?

5. In what time will $450 at 10 per cent. give $120 interest?

6. The interest on a note for $320 at 6¼ per cent. was $40 : for what time was the note drawn ?

7. In what time will $90 make $10.80 interest at 6 per cent. ?

8. In what time will $125 amount to $137.50 at 8 per cent. ?

9. In what time will $64 amount to $76.20 at 7 per. cent. ?

10. It required $1.520 to discharge a note drawn for 2⅓ years, at 7½ per cent. : find the face ?

11. In what time will a sum of money double itself at 8 per cent. ?

Sol.—It produces $\frac{2}{25}$ of itself $(=\frac{8}{100})$ in 1 year ∴ it produces a sum equal to itself in $1\div\frac{2}{25}$ years $=12\frac{1}{2}$ years.

Or, at 1 per cent. it doubles itself in 100 years ∴ at 8 per cent. it doubles itself in $\frac{1}{8}$ of 100 years$=12\frac{1}{2}$ years.

12. In what time will a sum of money treble itself at 6 per cent. ? at 8 per cent. ? at 7 per cent. ? at 10 per cent. ?

13. In what time will a given principal quadruple itself at 8 per cent. ? at 10 per cent. ? at 20 per cent. ?

14. The amount of a certain principal for a certain time, at 8 per cent., is $280, and at 5 per cent. the amount is $250 : find the time and the principal ?

Sol.—$280—$250=$30= interest at 3 per cent. for the required time ∴ interest at 5 per cent.$=\frac{5}{3}\times$ $30=$50 and $250—$50=

> \$200 the required principal, and \$200 produces, at 5 per cent., \$50 in 5 years, the required time.

15. The amount of a principal for a certain time at 6 per cent. was \$455, and at 8 per cent. the amount was \$490 : find the principal and the time.

16. The amount of a certain sum of money for a certain time at $7\frac{1}{2}$ per cent. is \$445, and the amount at 10 per cent. is \$460 : required, the time and the principal.

17. A sum of money on interest at 6 per cent. amounts to \$300, and at 10 per cent. for the same time it amounts to \$350 : find the time and the principal.

EXAMPLES.—15.

1. At what rate will \$120 in $2\frac{1}{2}$ years give \$21 interest?

> *Sol.*—At 1 per cent. for $2\frac{1}{2}$ years the interest $= \frac{1}{40}$ of principal $= \frac{1}{40}$ of \$120 $=$ \$3, and if \$3 come from 1 per cent., \$21 will come from $21 \div 3 = 7$ per cent.

2. At what per cent. will \$80 in $2\frac{1}{2}$ years give \$20 interest?

3. If the interest on \$450 for 3 years and 4 months is \$70, find the rate per cent.

4. At what rate per cent. will \$200 make \$49 interest in $3\frac{1}{2}$ years?

5. At what rate will \$300 make \$88 interest in 1 year and 10 months?

6. At what rate will $800 make $75 interest in 1 year and 3 months?

7. At what rate per cent. will $2,500 make $250 interest in 15 months?

8. At what rate will $6 amount to $7 in 3 years and 4 months?

9. At what rate will a given principal double itself in 10 years?

> *Sol.*—In 10 years it gains $\frac{1}{10}$ of itself at 1 per cent. ∴ In 10 years it gains $\frac{10}{10}$ (= 1) of itself at 10 per cent.

10. At what per cent. will a principal gain 2, 3, 4, 5, 6 times itself in twenty years?

11. At what rate per cent. will a given principal double itself in 10 years? 12 years? 20 years? 25 years?

12. At what rate will a principal become $2\frac{1}{2}$ times itself in 6 years? 8 years? 10 years?

13. The amount of a principal for 4 years at a certain rate per cent. is $620, and for 5 years $700: find the principal and the rate.

> *Sol.*—$700—$620=$80=interest on required principal for 2 (= 5—3) years ∴ $120 (= $\frac{3}{2}$ × $80) = interest for 3 years, and required principal = $620 — $120 = $500, and $500 gives $120 in 3 years, or $40 in 1 year ∴ rate = 8.

14. The amount of a principal for 4 years at a certain rate per cent. was $600, and for 6 years, at the same rate, the amount was $750: find the principal and the rate.

15. The amount of a certain principal for 7 years at a certain rate per cent. is \$540, and for 10 years, \$600 : find the principal and the rate per cent.

16. The amount of a principal for 4 years at a certain rate per cent, is \$420, and for 9 years the amount is \$570 : find the rate and the principal.

17. The amount of a certain principal for 3½ years at a certain rate per cent. is \$160, and for 4 years \$165 : find the principal and the rate per cent.

18 A note bearing interest for 200 days amounted to \$770 : find the rate per cent. Face \$730.

19*a*. I paid \$28.50 interest on a note drawn payable in 95 days : find the rate of interest. Face \$438.

19. At 12½ per cent. what would be the bank discount on \$72 for 4½ months ?

20. At 25 per cent. per annum, what would be the bank discount on \$24.96, for 9 months ?

21. What is the bank discount on \$75 for 4 months, at 13 per cent. ?

22. What is the bank discount on \$250 for 219 days, at 12 per cent. ?

23. Find the bank discount on a note of \$27.60, drawn at 70 days (allow the three days' grace), at 16⅔ per cent. ?

24. How much ready money can you get at the bank for a note of \$6.25, due in 3 months, discounted at 16 per cent ?

25. What amount must I mark on the face of a note drawn at 9 months, so that when discounted at 13⅓ per cent., I may get \$27.18 ?

26. A banker charged me \$7 discount on a note of \$140, due in 8 months: find the rate per cent.?

Sol.—\$7 for 8 months $=10\frac{1}{2}$ for 12, if 140 brings this, 100 will bring $7\frac{1}{2}$.

27. I pay \$14 discount on a note of \$250, at $9\frac{3}{5}$ per cent.: find the time for which the note was discounted.

28. The difference between the discount on a note and the cash received for it was \$264—the note being discounted for 5 months, at $14\frac{2}{5}$ per cent.: find the face of the note.

Section VII.—Discount (True).

EXAMPLES.—16.

1. Find the true discount on \$360 for three years four months, at 6 per cent.

Sol.—6 per cent. $\times\ 3\frac{1}{3}=20$ per cent. $=\frac{1}{5}$ ∴ present worth $+\frac{1}{5}$ of present worth $=360$, and present worth $=\$300$ ∴ disct. $=60$. Or, we have to add $\frac{1}{5}$ to p. w. to get the amount (\$360) ∴ we have to subtract $\frac{1}{6}$ from amount to get p.w., $\frac{1}{6}$ of $360=60$. *Note.*—It is seen that the *disct.*=int. on *present worth*.

2. Find discount on \$162 for 1 year 4 months, at 6 per cent.; and on \$58.58 for $1\frac{1}{2}$ years, at 8 per cent.

3. What is the present worth of \$112 for $1\frac{1}{2}$ years, at 8 per cent? what is the present worth of \$216, due in 2 years 11 months, at 12 per cent.?

4. The simple interest on principal for a certain time and rate is $\frac{1}{5}$ of itself: what fraction of it is the

Sol.—Since the interest of any sum is $\frac{1}{5}$ of itself ∴ the *present worth* of the principal $+\frac{1}{5}$ of this p.w.$=$the principal; *i.e.*, $\frac{6}{5}$ of p.w.$=$ the principal, and p.w.$=\frac{5}{6}$ of the principal ∴ $\frac{1}{6}$ of it is the discount.

5. If the simple interest is $\frac{1}{9}$ of the sum, what fraction is the true discount? if $\frac{1}{7}$? if $\frac{1}{12}$? if $\frac{1}{8}$? if $\frac{1}{20}$?

6. Find the discount on \$570, for 1 year 9 months, at 8 per cent. per annum.

Sol.—At simple interest \$100 would amount to \$114; thus \$114 gives \$14 discount, \$1 gives $=\frac{14}{114}=\frac{7}{57}$, and \$570 gives \$70.

7. The simple interest of a sum for a certain time and rate is $\frac{3}{8}$ of the sum: what fraction is the true discount?

Sol.—Present worth $+\frac{3}{8}$ of present worth$=\frac{11}{8}$ present worth, $=$ given sum ∴ present worth$=\frac{8}{11}$ given sum, and ∴ $\frac{3}{11}$ of given sum$=$discount.

8. If the simple interest is $\frac{3}{50}$ of principal, what is the true discount? if $\frac{2}{25}$? if $\frac{7}{50}$? if $\frac{a}{b}$?

9. If the true discount on a sum is $\frac{3}{43}$ of it, what is the simple interest? if $\frac{3}{20}$? if $\frac{7}{57}$? if $\frac{a}{b}$?

Sol.—$1-\frac{3}{43}=\frac{40}{43}=$present worth, and $\frac{3}{43}$ is the interest on $\frac{40}{43}$ ∴ interest on 1 is $\frac{3}{43}\div\frac{40}{43}=\frac{3}{40}$ (*Note*, $=\frac{3}{43-3}$).

10. What fractional part of a note bearing interest at 20 per cent. would you have to add to it in order to find its value in one year hence?

11. If by adding $\frac{1}{5}$ to its face you get the amount of a note one year hence, how much must you take off the amount of a note due in one year to find its present value?

12. If by adding on 30 per cent., or $\frac{3}{10}$, you get the amount of a note, how much taken off would reduce it again to its present value?

13. How does the numerator of what is taken off compare with the numerator of what is added?

14. How does the denominator of the fractional part taken off compare with the numerator and the denominator of what is added?

15. If adding $\frac{2}{5}$ of a note of $15 to it make it worth $21 in one year, what fractional part would you have to take off a note worth $21 in one year to find its true discount, and what would its true discount be?

16. Find the true discount on a note of $54, due in 6 months, at 16 per cent.

17. Find the true discount on a note of $153, due in 2 months, at 12 per cent.

Sol.—Interest$=\frac{1}{50}$ ∴ discount $=\frac{1}{51}=\frac{1}{51}$ of 153 $=$\$3.

18. Find the present value of a note of $164 due in 4 months, at $7\frac{1}{2}$ per cent.

19. If the true discount for one year is $5, and the amount of the note $15, find the rate per cent.

20. If $\frac{1}{4}$ taken off gives the bank discount for a certain time and date, what taken off would give the true discount?

21. What is the difference between the bank discount and the true discount, at $33\frac{1}{3}$ per cent., for 1 year? for 2 years?

Sol.—For 1 year, interest$=\frac{1}{3}$ ∴ discount$=\frac{1}{4}$ of principal; for 2 years, interest$=\frac{2}{3}$ of principal ∴ discount$=\frac{2}{5}$; difference in former case$=\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$, &c.

22. What is the difference between the bank discount and the true discount, at 20 per cent., for 1 year? for 3 years?

23. Find the difference between the bank discount and the true discount on a note of $126, at $16\frac{2}{3}$ per cent., for 1 year; for 2 years.

24. Find the difference between the bank discount and the true discount, on a note of $36, at $12\frac{1}{2}$ per cent., for 1 year; for 2 years.

25. Find the difference between the bank discount and the true discount, on a note of $1,640, due in 4 months, at $7\frac{1}{2}$ per cent.

26. If $4 is the difference between the bank discount and the true discount, at 10 per cent. for 1 year, what was the amount of the note?

27. If the rate per cent. is 20, what fraction will the true discount be of the bank discount, for 1 year? for 3 years?

28. If the true discount for a year is $\frac{7}{8}$ of the bank discount, find the rate per cent. What will be the ratio of the discount to the interest for two years?

29. If the true discount for 6 months is $10, and the bank discount $11 for the same time, find the rate per cent. and the amount of the note.

30. Find the rate per cent. and the time, if the bank discount of a note of $120 is $8.10, the rate per cent. being equal to the number of months for which the note was drawn.

Sol.—\$8.10 is found to be $\frac{81}{1200}$ of principal: At 1 per cent. per annum, the int. for 1 month would be $\frac{1}{1200}$, ∴ at 9 per cent. per annum, the interest for 9 months would be $\frac{81}{1200}$. *Ans.* 9 per cent. for 9 months.

31. If \$13 is the bank discount on a note of \$65 for 8 months, what would be the true discount on the same note for 1 year?

32. The simple interest on a sum of money, for a certain time and rate is \$150, and the true discount is \$120: find the sum of money.

33. I lent a man \$100 for the year 1877, and at the end of the year he returned it, and lent me \$100 for the year 1878, expecting neither of us would have any advantage over the other. How much better off would I be at the end of 1878 if I had invested my \$100 money being worth 10 per cent.?

34. I have a \$50 note due in 6 years, not bearing interest: what would I get for it at the bank, discounted at 10 per cent.?

35. I have two notes (1 year to run) which are worth \$38, the one I get discounted at a bank, the other by true discount; the whole discount paid is \$7, rate in both cases 20 per cent.: find the face of the note discounted at the bank.

Sol.—Interest = $\frac{1}{5}$ discount = $\frac{1}{6}$: bank discount on *both* notes would have been $\frac{1}{5}$ of \$38 = \$7$\frac{3}{5}$; \$7$\frac{3}{5}$ − \$7 = $\frac{3}{5}$ loss: \$1 gives loss of $\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$. What will give loss of \$$\frac{3}{5}$? $\frac{3}{5} \div \frac{1}{30}$ = \$18 = amount of note on which true discount was taken ∴ 38 − 18 = \$20 = do. bank discount.

35*a*. I had two notes (15 months to run) amounting to \$210 ; both are discounted at 10 per cent., one at bank discount, the other at true discount—the entire discount being \$25 : find the face of the note on which true discount was allowed.

36. The interest on a certain sum for a certain time is $\frac{1}{19}$ of the principal : what fraction of the principal is the discount for the same time and rate ?

37. The interest is $\frac{2}{5}$ of the principal, and the difference between the interest and discount is \$8 : find the principal.

38. The interest is \$5, the discount for same time and rate is \$4 : what is the sum ?

39. In what time, at 8 per cent. simple interest, will the amount be $1\frac{4}{5}$ of the principal ?

40. The discount on a certain sum for two years is $\$2\frac{8}{11}$, and the interest for the same time and rate is \$3 : find sum and rate per cent.

41. The interest is \$40, and the interest on the discount for same time and rate is $\$3\frac{7}{11}$: find the discount.

42. The interest is \$2, and the difference between the interest and discount is $16\frac{2}{3}$ cents : find the principal.

> *Sol.*—$\$2 - \$\frac{1}{6} = \$\frac{11}{6}$ = discount and $\$\frac{1}{6}$ is interest on $\frac{11}{6}$ ∴ interest is $\frac{1}{6} \div \frac{11}{6} = \frac{1}{11}$ of principal ∴ $\$2 \times 11 = 22$, the required principal.

43 \$200 has \$40 for its interest : find discount for same time and rate.

44. A horse that cost \$90 is offered for sale on a credit of 12 months: what price should the seller ask so that he may fall $9\frac{1}{11}$ per cent. of his asking price and still have $11\frac{1}{9}$ per cent. profit, money being worth 10 per cent.?

45. A man bought an article for $106 on 12 months' credit and sold it on 9 months' credit, so as to neither gain nor lose: find his selling price, money being worth 6 per cent.

46. A note drawn at 9 months at 8 per cent. interest, is discounted at a bank 6 months before maturity: what fraction of the face of the note must the bank give for it to make 10 per cent. on its money?

Sol.—At 8 per cent for 9 months the note will amount to $\frac{53}{50}$ of its face, banker is to make $\frac{1}{20}$ of his outlay in 6 months $\therefore$ $\frac{21}{20}$ of outlay $= \frac{53}{50}$ of face of note, and outlay$=$ $\frac{20}{21}$ of $\frac{53}{50} = \frac{106}{105}$ of face of note.

47. A note of $200 drawn for six months at 8 per cent. per annum is discounted by a broker 4 months before it is due: what must the broker pay for it in order to make 12 per cent. per annum on his money?

CHAPTER II.

VARIOUS RULES.

Section I.—Ratio and Proportion.

EXAMPLES.—17.

1. What part of 4 is 2?

Sol.—1 is $\frac{1}{4}$ of 4 $\therefore$ 2 is $2\times\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$. In this case $\frac{1}{2}$, that is, the *quotient* of 2 by 4, is called the *ratio* of 2 to 4, which is sometimes written 2 : 4.

2. What is the ratio of 3 to 6? 4 to 6? 9 to 12? $10 to $12? 600 to 800?

3. What part of $\frac{4}{5}$ is $\frac{2}{3}$?

> *Sol.*—$\frac{4}{5}=\frac{12}{15}$, and $\frac{2}{3}=\frac{10}{15}$, and the ratio of 10 units of any kind to 12 units of the same kind is $\frac{10}{12}=\frac{5}{6}=\frac{2}{3}\div\frac{4}{5}$, as in the former case.

4. What is the ratio of $\frac{3}{4}$ to $\frac{1}{5}$? $\frac{1}{2}$ to $\frac{9}{10}$? $\frac{6}{7}$ to $\frac{5}{8}$?

5. What is the ratio of $2\frac{1}{2}$ to 3? $3\frac{1}{3}$ to $2\frac{1}{2}$? $\frac{8}{9}$ to 2? 6 to $\frac{3}{8}$?

6. What is the ratio of $5 to $25? 3*s.* 6*d.* to 2*s.* 9*d.*? 1 bush. 1 pk. to 12 qts.?

7. What is the ratio of 1ft. 6in. to $1\frac{1}{2}$ yds.? of $4\frac{1}{2}$ acres to $\frac{1}{3}$ of $3\frac{1}{4}$ acres?

8. Compare the rates at which two men travel per hour, one of whom goes $2\frac{1}{2}$ miles in one hour, the other $3\frac{1}{3}$ miles in one hour.

9. One steamer makes $8\frac{1}{2}$ miles in 40 minutes, and a second makes 9 miles in 45 minutes: compare their rates per hour.

10. To what number has 6 the same ratio that 3 has to 21?

> *Sol.*—$3:21=\frac{3}{21}=\frac{1}{7}$, so the question really is—Of what number is 6 the $\frac{1}{7}$? 6 is $\frac{1}{7}$ of 42 $\therefore$ 3 to 21 has the same ratio that 6 has to 42. (*Note that* $42=\frac{21\times6}{3}$; 7 is called the "*fourth* proportional" to the other three. Let teacher show relation between extremes and means.)

11. To what has 7 the same ratio that 4 has to 32?

12. To what has $\frac{1}{2}$ the same ratio that 2 has to 4?

13. Five shillings has to 7*s.* 6*d.* the same ratio that $8 has to what?

14. To what has six hours the same ratio that 3 men has to five men?

15. To what has 1 acre the same ratio that 15 dwt. has to $2\frac{1}{2}$ oz.?

16. Compound the ratio 3:4 and 8:12.

> *Sol.*—$3:4=\frac{3}{4}$; $8:12=\frac{8}{12}$; and $\frac{3}{4}\times\frac{8}{12}=\frac{24}{48}$ $=24:48$ (or $=1:2$), what is the ratio compounded of the two given ratios?

17. Compound the ratios 4:5 and 7:8; also $\frac{5}{9}$ and $\frac{2}{3}$.

18. The ratio of A:B is 3:4, and of B:C it is 4:5; find the ratio of A:C.

19. One milkman adds 3 pints of water to every $1\frac{1}{2}$ gallon of milk, another adds 5 pints to every $2\frac{1}{2}$ gallons: compare the quantity of water in the two samples of milk.

20. The ratio of A:B is $\frac{1}{2}$ of B:C 3:4, of A:D 4:5: find the ratio of A:D.

21. If 4 melons are worth 12 oranges, and 5 oranges are worth 15 cents, how many cents are 12 melons worth?

22. What number bears to 6 the same ratio that 6 bears to 9?

> *Sol.*—$6:9=\frac{2}{3}$.·. the required no. $=\frac{2}{3}$ of $6=4$. We have then 9:6::6:4.
>
> *Note.*—Six is called the "mean proportional" between 9 and 4: $6^2=449$.·. $6=\sqrt{4\times 9}$.

23. Find the mean proportionals between 8 and 2, 4 and 16, 5 and 20, 9 and 16.

EXAMPLES.—18.

1. If 5 acres cost $75, what will 8 acres cost?

> To the cost of 8 acres $75 will have the ratio of 5:8; *i.e.*, $\frac{5}{8}$ of some number is 75, what is the number? $\frac{5}{8}$ is 75, $\frac{1}{8}$ is 15, and $\frac{8}{8}$ is $8\times15=120$.

2. Ten pounds of sugar cost 95 cents: what will 8 cost?

3. A man digs a ditch in 20 days, working 10 hours a day: how long will it take him if he works 8 hours a day?

4. It costs $60 to carpet a room 15 feet wide, and 18 feet long: how much would it cost if the room were 16 ft. 6 in. long?

5. John earns $7 as often as James earns 10\frac{1}{2}$: when James has earned 52\frac{1}{2}$ how much has John earned?

6. If $\frac{3}{8}$ of a yard of cloth cost $$\frac{7}{8}$, how much will 1$\frac{7}{8}$ yds. cost?

7. Two persons travel towards each other; the first travels 6 miles while the second travels 5: when they meet it is found that the first has travelled 51 miles: how far were they apart when they started?

8. If 5 men receive $75 wages in 10 days, how much would 15 men receive in 40 days at the same rate?

> *Sol.*—In 10 days 15 men would receive $\frac{15}{5}\times75=\$225$ ∴ in 40 days 15 men would receive $4\times225=\$900$.

9. If $150 gain $12 in 12 months, how long will it take $600 to gain $18?

10. If 6 horses eat 9 tons of hay in 8 months, how many tons would 5 horses eat in 12 months?

Sol.—In 12 months 6 horses would eat $13\frac{1}{2}$ tons $\therefore$ 5 would eat $\frac{5}{6}$ or $13\frac{1}{2}=\frac{45}{4}=11\frac{1}{4}$.

11. If the freight on 8 cwt. for 36 miles is $8, what would it be for 20 cwt., 18 miles, at the same rate?

12. If 12 oz. of bread can be bought for 6 cents when flour is $6 a barrel: how much should be bought for 8 cents when flour is $4 a barrel?

13. How many men in six days of 10 hours each can earn as much as 6 men in 20 days of 8 hours each?

14. If 4 men can do a work in 6 days, in what time will it be completed if they receive the assistance of 10 men, when one-fourth of the work is done?

15. If 6 men can do the work of 18 women, and 2 women can do the work of 3 boys, how many men can do the work of 36 boys?

Sol.—It will take $\frac{36}{3}\times 2=24$ women $\therefore$ it will take $\frac{24}{18}\times 6=8$ men.

16. How many sheep can you buy for 3 horses, if 10 cows are worth 5 horses, and 6 sheep are worth 2 cows?

17. How many oranges can you buy for 20 cents, if 4 oranges are worth 8 apples, and 6 apples are worth 12 cents?

18. If 5 lbs. of cheese are equal in value to 2 lbs. of butter, and 6 lbs. of butter to 2 bushels of oats, how many lbs. of cheese will pay for 8 bushels of oats?

19. If 9 men build 10 rods of wall in 8 days, in what time can 20 rods be built, if $\frac{2}{3}$ of their number leave when the work is half done?

Section II.—Proportional Parts.

EXAMPLES.—19.

1. Divide $35 into two parts which shall be in the ratio of 2 : 3.

> *Sol.*—5 thus divided would give the numbers 2 and 3 $\therefore$ 35 thus divided would give the numbers 7×2 and 7×3, or 14 and 21.

2. Divide 21 acres between A and B in the ratio of 3 : 4.

> *Sol.*—Of 7 acres A gets 3 and B 4 $\therefore$ A is to get $\frac{3}{7}$ of the whole, and B $\frac{4}{7}$ of the whole, or 9 and 12 respectively.

3. Divide 48 into two parts which shall have the ratio 5 : 7.

4. Divide 144 into parts which shall be as 3 : 5, as 4 : 5, as 5 : 7, as 7 : 11, as 11 : 13.

5. Divide $70 between Harry and Willie, so that as often as Harry gets $4 Willie may receive $3.

6. $45 is divided between two boys, so that as often as the first gets $\$\frac{1}{2}$ the second gets $\$\frac{2}{5}$: find their shares.

7. 60 apples are divided among three boys, so that their shares may be in proportion to 3, 4, 5.

> *Sol.*—Of 12 the first would get 3, the second 4, and the third 5 $\therefore$ of 60 the first would get 5×3, the second 5×4, and the third 5×5.

8. Three men bought 75 horses, and as often as the first paid \$4, the second paid \$5, and the third 6: how many horses should each receive?

9. The sum of two numbers is 105, and the first is to the second as $\frac{1}{2} : \frac{3}{4}$: find the numbers.

10. Three men went into a speculation, contributing \$250, \$300, \$350 respectively; they gain \$1,350: how should this be divided?

11. Three men do a certain work for \$60, they work the same number of days, and are to receive \$1, \$1.25, and \$1.50 a day respectively: how should the money be divided?

12. Divide \$900 among A, B, C, so that B may get twice as much as A, and C three times as much as B.

> *Sol.*—B's share is 2 times A's, and C's = 3 times B's = 6 times A's .·. A's + B's + C's = 1 + 2 + 6 = 9 times A's share = \$900 .·. A's = \$100, B's \$200, C's \$600.

13. In an orchard are 96 trees; there are 5 apple trees for every 4 peach trees, and 4 peach trees for every 3 pear trees: how many trees of each kind are there?

14. Three men hired a pasture for \$30; A put in 3 horses, B 4 horses, and C 5 horses: how much should each pay?

15. Divide \$340 among A, B, C, in the proportion of 1, $\frac{2}{3}$, $\frac{3}{10}$.

16. Divide \$125 among 2 men, 3 women, and 4 boys, so that as often as each boy gets \$2, each woman may get \$3, and each man, \$4.

17. $\frac{1}{5}$ of the time past noon is $\frac{1}{3}$ of the time till midnight: what o'clock is it?

Sol.—Time from noon till midnight = 12 hours
∴ Divide 12 in ratio of $\frac{1}{3} : \frac{1}{5}$, *i.e.*, 5 : 3.
Or, $\frac{1}{5}$ of time past noon = $\frac{1}{3}$ time till midnight.
∴ time past noon = $\frac{5}{3}$ time till midnight, &c.

18. What o'clock is it when the time past noon is $\frac{2}{5}$ of the time till midnight?

19. A's fortune added to $\frac{1}{2}$ of B's equals \$2,000; and A's is to B's as 3 : 4: find the fortune of each.

20. I bought two lots, the prices of which were as 9 : 8; $\frac{1}{3}$ of price of first lot added to $\frac{1}{4}$ of that of second equals \$500: find the price of each.
for $2\frac{1}{2}$ years at $7\frac{1}{2}$ per cent. : find the face?

EXAMPLES.—20.

1. John's age being doubled, and increased by $\frac{3}{4}$ of his age + 10 years, equals 50: how old is he?

2. John's age is to Richard's in the ratio of $\frac{3}{4}$ to $1\frac{1}{4}$, and the sum of their ages is 32: find the age of each.

3. What number is that which being increased by the difference between its fourth and fifth parts, equals 42?

4. The sum of two numbers is 22, and $\frac{2}{3}$ of the first equals $\frac{4}{5}$ of the second: find the numbers.

Sol.—The first = $\frac{4}{5} \div \frac{2}{3} = \frac{6}{5}$ of second.
∴ second + $\frac{6}{5}$ of second = $\frac{11}{5}$ of second
= 22, and second is 10, first 12.

5. Mary gave $\frac{3}{4}$ of her money in charity, saved $\frac{2}{5}$ as much as she had given away, and then had \$60: how much had she at first?

6. A, B, and C are worth together $1,800; A is worth $\frac{2}{3}$ as much as B, and C $\frac{1}{2}$ as much as A: how much is each worth?

7. $500 is divided among 4 men; the first has $\frac{1}{4}$ as much as the second, the second $\frac{1}{2}$ as much as the third, and the third $\frac{2}{3}$ as much as the fourth: how much does each receive?

8. Find two numbers whose sum is 121, and which have the ratio of $\frac{5}{8} : \frac{3}{4}$.

9. A boy spent $\frac{2}{5}$ of his money, and then found $15 was $\frac{5}{8}$ of what he had remaining: how much had he at first?

Sol. $\frac{2}{5}$ spent $\therefore$ $\frac{3}{5}$ left; $\frac{5}{8}$ of this $= \frac{3}{8} =$ $15.
$\therefore$ $40 at first.

10. If $\frac{2}{5}$ of a number be multiplied by $2\frac{1}{8}$, and 70 be taken from the product, the result is $\frac{1}{2}$ the number: find the number.

11. A horse and harness cost $150, and $\frac{4}{5}$ of the cost of the horse was $3\frac{1}{5}$ times the cost of the harness: find the cost of each.

12. A calf, a cow, and a colt were sold for $110, the colt brought $15 less than the cow, and the calf $25 less than the colt: what did each bring?

Sol.—Colt brought $15 less than the cow, and the calf $25 less than the colt, *i.e.*, $40 less than the cow $\therefore$ 3 times price of cow, less $55 = $110; and 3 times price = $110 + $55 = $165, &c.

13. Divide 200 into three such parts that the second shall be 4 times the first, and the third 5 times the second.

14. Two farmers have 165 sheep; one has 5 less than $3\frac{1}{4}$ times what the other has: how many has each?

15. I sold $\frac{2}{5}$ of my farm, afterwards bought 40 acres, and had then 130 acres: how much had I at first?

16. Three lots cost \$500, the second cost \$50 more than the first, and the third \$100 more than the second: find the cost of each.

17. Find two numbers whose difference is 20, and which have the ratio 5 : 4.

Sol.—The less is plainly $\frac{4}{5}$ of greater $\therefore$ difference $= \frac{1}{5}$ of greater $= 20$ and greater $=$ 100.

18. My age now is to my age 10 years ago as 9 : 7: find my present age.

19. A cow and a horse cost \$165; the cow cost \$10 less than $\frac{2}{5}$ of what the horse cost: find the cost of each.

20. Divide 324 into 3 such parts that $\frac{1}{2}$ of the first shall be twice the second, and $\frac{1}{4}$ of the second 4 times the third.

21. $\frac{3}{5}$ of A's age 4 years ago is $\frac{3}{7}$ of what it will be 4 years hence: find his age.

Section III.—Partnership.

Examples.—21.

1. Three partners put into a business, \$200, \$300, \$400: they gain \$1,350: how should this be divided?

Sol.—\$900 gains \$1350, 1 gains $1350 \div 900 =$

\$$1\frac{1}{2}$ ∴ the shares are $200 \times 1\frac{1}{2}$, $300 \times 1\frac{1}{2}$ $400 \times 1\frac{1}{2}$: 16; \$300, \$450, \$600; Or:—the amounts of capital are as 2, 3, 4 ∴ the first gets $\frac{2}{9}$ of the gain, the second $\frac{3}{9}$, and the third $\frac{4}{9}$.

2. A owes B \$200, C \$300, and D \$400; his property is worth only \$450: how much do B, C, D, each receive?

3. A, B, C, hire a pasture for \$80; A put in one cow 5 months; B 2 cows 3 months; C 3 cows 3 months: how much should each pay?

Sol.—1 cow 5 months = 5 cows for 1 month; 2 cows 3 months = 6 cows for 1 month; 3 cows 3 months = 9 cows for 1 month ∴ 5, 6, 9 are the proportions in which the cost must be paid $5 + 6 + 9 = 20$ ∴ $\frac{5}{20}$ of 80, $\frac{6}{20}$ of 80, $\frac{9}{20}$ of 80, or \$20, \$24, and \$36, are the amounts.

4. Four men in partnership divide their stock into 40 shares; A has 6 shares, B 8, C 12, and D 14; they gain \$15,000: how should this be divided?

5. A and B are partners, A furnished \$500 for 6 months, and B \$750 for 8 months; they gain \$1500: how should this be divided?

6. The stock of a bank is divided into 1500 shares; when it makes a dividend of \$7,500, how much does A receive who owns 19 shares?

7. A puts in \$250 for 6 months, and B \$350 for 4 months; they gain \$580: how should this be divided?

8. A, B, C form a partnership; A puts in $\frac{1}{4}$, B $\frac{5}{8}$, and C the remainder: C's gain is \$240: what does each of the others gain?

9. A and B are partners, A put in $\frac{5}{12}$ of the stock, and B the remainder; B's gain was $1400: find A's.

10. A and B are partners, A puts in $\frac{2}{5}$ of the stock for 4 months, and B the remainder for 3 months: how should a gain of $1700 be divided?

> *Sol.*—$\frac{2}{5}$ for 4 months $= \frac{8}{5}$ for 1 month, and $\frac{3}{5}$ for 3 months $= \frac{9}{5}$ for 1 month $\therefore$ gain is to be divided in rates 8 : 9; $800, $900

11. A and B ship a cargo of coal from Pictou to Montreal, agreeing to share gains and expenses equally. A paid $80 for loading and freight. The coal was sold in Montreal for $680. B had previously lent A $100 to purchase coal, and he also owed A $20 on a previous shipment; how should the proceeds of the coal be divided?

12. A rented a house for one year, agreeing to give $6 a month, and pay the taxes for the year; he remained in the house 3 months, and during that time paid the taxes. If the rent of the house paid the owner 9 per cent. on his money, and the assessment in the locality was $1\frac{1}{2}$ cents on the dollar: what, in justice, should A pay for his 3 months rent?

13. Smith and Jones hire a horse for $5, to go from Truro to Windsor, a distance of 40 miles. Having driven 10 miles they took up Brown, brought him to Windsor and back to where they found him: how much of the horse hire should each man pay?

14. A, B, and C are partners, whose respective shares of the stock are as 1, 2, and 3. They gain $600. If, on dividing the profits, A relinquish his part of the gain, how much will each of the others receive?

15. A and B start business. A put in $30 for 4 months, and B a certain sum for two months, they gain $44, of which A takes $24 for his share: find the sum B puts in.

> *Sol.*—A's stock × time = 30 × 4 = 120; the gains are as 24 : 20; or 6 : 5 ∴ $\frac{5}{6}$ of 120 = product of B's stock by time, which ∴ = $100 ∴ $100 ÷ 2 = $50, B's stock.

16. An old man agreed to thresh pease for a farmer and take the tenth bushel for his pay; the farmer, who could thresh as much in 5 hours as the old man in 7, worked with him: what bushel of the combined threshing should the old man get?

17. Two men, A and B, rent a pasture for $21.50. A put in 2 cows, and B 3 horses. Supposing a horse to eat half as much again as a cow in the same time, but the cows to be on the grass one-third as long again as the horses, find what each man should pay.

18. A and B invest capital in the proportion of 1 : 2, at the end of 2 months they each withdraw ½ the sum invested, at the end of the year the gain is found to be $210: how should this be divided.

19. Three men, A, B and C, enter into partnership for the purpose of renting a skating rink at $90. A put in his money for 3 months, B his for 1 month, and C his for 2 months. The earnings of the rink for the season being divided, A got $30, B $15, and C $20: how much money did each man put in?

20. A and B do a piece of work for $16, they agree to divide the money in proportion to their ability to work, which is as 1 to 2, and also to the time each worked which is as 2 to 3: how should the money be divided?

21. A and B form a partnership; A puts in \$200 for a certain time, B \$300 for 2 months, B's share of the gain is \$450, and A's \$600: how long was A's money in the business?

> *Sol.*—A's share $= \frac{4}{3}$ B's $\therefore$ A's product of stock by time $= \frac{4}{3}$ of $300 \times 3 = 800$, and $800 \div 200 = 4 =$ A's time.

22. A and B are partners; A puts in \$150 for 6 months, and B \$250 for a certain time; the profits are divided in the ratio of 9 : 10: find the time B's stock was in.

23. A and B are partners; A contributes $\frac{2}{3}$ of the joint capital for 5 months, and B receives $\frac{6}{11}$ of the gain: find B's time of investment.

24. How much copper ore was raised to produce 500 tons of metal, allowing $\frac{3}{8}$ of the ore to be lost in roasting, and $\frac{1}{3}$ of the remainder in smelting?

25. In what proportion must water be mixed with milk, to reduce the value from 20 cents a gallon to 16 cents?

> *Sol.*—The milk in the mixture is worth 16 cts., $\therefore$ there must be $\frac{16}{20}$ $(= \frac{4}{5})$ of gallon, and hence $\frac{1}{5}$ gallon of water $\therefore$ the ratio is 1 : 4.

26. A grocer has 60 lbs. of mixture of coffee and chicory, the former to the latter as 5 : 1: how much chicory must be added to make the ratio 4 : 1?

Section IV.—Alligation.

EXAMPLES.—22.

1. A milkman sells milk at 16 cents a gallon: in what proportion must he add water so as to reduce the price to 14 cents a gallon?

> *Sol.*—1 gallon milk sold at 14 cts. gives *loss* 2 cts; 1 gallon water sold at 14 cts gives gain 14 cts. ∴ $\frac{1}{7}$ gal. water gives gain 2 cts., which balances the 2 cts. loss ∴ ratio is $\frac{1}{7}$: 1, or 1:7.

2. How much water per gallon must be added to brandy worth $4 a gallon, to reduce the price to $3 a gallon?

3. In what proportion must wines worth 10*s.* and 14*s.* a gallon respectively, be mixed, so that the mixture may be worth 13*s.* a gallon?

> *Sol.*—By selling 1 gal. 10*s.* quality at 13*s.* there is gain of 3*s.*; by selling 1 gal. 14*s.* quality at 13*s.* there is loss of 1*s.*, and 3 of latter gives loss of 3*s.*, which balances gain of 3*s.* ∴ ratio is 1:3.

4. How much wine at 14*s.* a gallon and 18*s* must be taken to form mixture worth 17*s.*?

5. How much sugar at 6 cts. and 10 cts. per lb. must be taken to form a mixture worth 7 cts?

6. How much chicory at 10 cts. a lb., and coffee at 30 cts., must be taken to make 20 lbs. worth 25 cents per lb.?

Sol.—Find first the *proportion* in which they are to be mixed; 1 lb. chicory sold at 25 cts. gives *gain* 15 cts., 1 lb. coffee sold at 25 cts. gives *loss* 5 cts. Hence 3 lbs. of coffee must be taken to 1 of chicory ∴ dividing 20 lbs. (required mixture), in ratio of 1 : 3, we get 5 lbs. chicory and 15 lbs. coffee.

7. How much chicory at 8 cts. and coffee at 24 cts. must be mixed to get 40 lbs. of a mixture worth 22 cents a pound?

8. How much sugar at 6 cents a pound must be mixed with 30 lbs. at 10 cents to get a mixture worth 7 cts.?

Sol.—Find first the prop. in mixture. As before, this will be found to be 3 lbs. at 6 cts., and 1 lb. at 10 cts., but we require 30 lbs. at 10 cts. ∴ 30×3=90 lbs. at 6 cents.

9. How much water must be mixed with 10 gallons of milk worth 20 cts. a gallon, to get a mixture worth 18 cts. a gallon?

10. How much sugar at 6 cts., 8cts. and 10 cts. per lb. must be taken to form a mixture worth 9 cts. per lb.?

Sol.—1 lb. of 6 ct. sold at 9, gives 3 cts. gain; 1 lb. 8 ct. sold at 9 cts. gives 1 ct. gain—total gain, 4 cts.; 1 lb. 10 ct. sold at 9 gives 1 ct. loss ∴ 4 at 10 gives 4 cts. *loss* to balance 4 cts. gain, and proportions are 1, 1, 4.

11. In what proportions must flour worth 2 cts., 3 cts., 5 cts. per lb. respectively, be taken so as to get a mixture worth 4 cts. per lb.?

12. In what proportions must wine worth 10*s*., 14*s*., 18*s*. a gallon, be taken to form a mixture worth 16*s*.?

13. A grocer mixed sugar at 6 cts. and 12 cts., and by selling the mixture at 8 cts., he gained 25 per cent. on the first kind, and $33\frac{1}{3}$ per cent. on the second kind: in what proportion did he mix them?

Sol.—Six cts. + 25 per cent. of it $= 7\frac{1}{2}$; 12 cts. + $33\frac{1}{2}$ per cent. of it = 16 cts. 1 lb. of $7\frac{1}{2}$ sold at 8, gains $\frac{1}{2}$ ct.: 1 lb. of 16 sold at 8 loses 8, $\therefore$ ratio is $8 : \frac{1}{2} = 16 : 1$.

14.—A grocer mixed coffee at 20 cts. a lb. with another quality worth 35 cts. a lb., and by selling the mixture at 30 cts. a lb. he gained 25 per cent. on the former, and 20 per cent. on the latter: in what proportion did he mix them?

Section V.—Special Methods.

EXAMPLES.—23.

1. Find the square of 37.

Sol.—$37^2 = (3 \text{ ten} + 7)^2 =$ (by actual multiplication) $9 \text{ ten}^2 +$ twice $7 \times 3 \text{ ten} + 49 = 90 \text{ ten} + 42 \text{ ten} + 4 \text{ ten} + 9 = 136 \text{ ten} + 9 = 1369$.

2. Find the square of 206 $(= 20 \text{ ten} + 6)$.

Sol.—$206^2 = 400 \times 10^2 + 240 \times 10 + 36 = 4243 \times 10 + 6 = 42{,}436$.

3. Square 196 $= (20 \text{ ten} - 4)$.

Sol.—$(196)^2 = (20 \text{ ten} - 4)^2 = 400 \times 10^2 - 160 \times 10 + 16 = 4{,}000 \text{ ten} - 160 \text{ ten} + 16 = 3{,}840 \text{ ten} + 16 = 38{,}416$.

4. Find squares of 116, 105, 195, 304, 299.

5. Write down the squares of 197, 711, 801, 795, 497, 509, 805.

NOTE.—When the number is less than 100, a convenient rule, derived from the rule for squaring as above, is : Add the units to the number, multiply by the tens, and add the square of the units, *e.g.*:

6. Square 35.

Sol.—$(35+5) \times 30 + 5^2 = 1225$.

7. Find the squares of 25, 45, 65, 84, 85.

8. Find the squares of 36, 39, 72, 85, 89.

9. Find the squares of 69, 58, 23, 87, 61.

10. Square $8\frac{1}{2}$.

Sol.—$(8\frac{1}{2})^2 = (8+\frac{1}{2})^2 = 8^2 + 2 \times \frac{1}{2} \times 8 + \frac{1}{4}$
$= 8^2 + \textit{one} \times 8 + \frac{1}{4} = 8\,(8+1) + \frac{1}{4}$
$= 72\frac{1}{4}$.

11. Find the squares of $3\frac{1}{2}$, $5\frac{1}{2}$, $7\frac{1}{2}$, $10\frac{1}{2}$, $12\frac{1}{2}$.

12. Find the squares of $14\frac{1}{2}$, $19\frac{1}{2}$, $24\frac{1}{2}$, $29\frac{1}{2}$.

13. Find the square of $8\frac{1}{4}$.

Sol.—$(8\frac{1}{4})^2 = (8+\frac{1}{4})^2 = 8^2 + \frac{1}{2} \times 8 \times \frac{1}{16} =$
$68\frac{1}{16}$.

14. Find the squares of $4\frac{1}{4}$, $6\frac{1}{4}$, $10\frac{1}{4}$, $12\frac{1}{4}$.

Find the squares of $3\frac{1}{4}$, $5\frac{1}{4}$, $20\frac{1}{4}$, $16\frac{1}{4}$, $14\frac{1}{4}$.

15. Multiply 37 by 35 (*i. e.* 3 ten + 7 by 3 ten + 5).

Sol. (3 ten + 7) (3 ten + 5) = 90 ten +
(7+5) × 3 ten + 35 = 90 ten + 36 ten
+ 3 tens + 5 = 129 ten + 5 = 1,295.

16. Find the products of 34 and 37, 32 and 39, 45 and 47, 56 and 54, 63 and 67.

17. Find the product of $6\frac{1}{4}$ by $6\frac{3}{4}$.

Sol.—$6\frac{1}{4} \times 6\frac{3}{4} = (6 + \frac{1}{4})\,(6 + \frac{3}{4}) = 6 + 1$
$\times 6 + \frac{1}{4} \times \frac{3}{4} = 6\,(1 + 6) + \frac{3}{16} = 42\frac{3}{16}$.

18. Find the products of $5\frac{1}{4}$ and $5\frac{3}{4}$, $7\frac{1}{4}$ and $7\frac{3}{4}$, $7\frac{1}{5}$ and $7\frac{4}{5}$, $8\frac{3}{7}$ and $8\frac{4}{7}$, $10\frac{1}{5}$ and $10\frac{4}{5}$.

19. Find the number of square rods in a lot which is 32 rods square.

20. Find the cost of $9\frac{3}{4}$ lbs. of sugar at $9\frac{1}{4}$ cents per lb.

21. What will $5\frac{5}{8}$ cords of wood come to at \$$5\frac{3}{8}$ per cord?

22. Find the cost of $15\frac{1}{2}$ yards of print at $15\frac{1}{2}$ cts. per yard.

23. What will $5\frac{4}{5}$ yards of broadcloth cost at $5.20 a yard?

EXAMPLES.—24.

1. Multiply 237 by 14.

[*]*Sol.*—Multiply each figure of multiplicand by the units of multiplier, and add the figure to the right of the one multiplied: $4 \times 7 = 28$, write down 8; $4 \times 3 + 7$ (+ 2 carried) = 21, write down 1; $4 \times 2 + 3 + 2$ (carried) = 13, write down 3; then 2 + 1 (carried) = 3, write down 3 ∴ product is 3,318.

2. Find product of 345 by 17, 39 by 19, 121 by 18. Write down product of 632 by 15, 32,217 by 13, 479,632 by 14, 215,762 by 18, 31,451 by 19, 3,474,263 by 17.

3. Multiply 4,347 by 31.

Sol.—Write down the units figure of multiplicand; multiply by the tens of the multiplier, adding to each result the figure to the left of the one multiplied, *e.g.*: Write down 7; $3 \times 7 + 4 = 25$, write down 5; $3 \times 4 + 3 + 2$ carried $= 17$, write down 7; $3 \times 3 + 4 + 1$ carried $= 14$, write down 4; $3 \times 4 + 1$ carried $= 13$, write down 3; $+ 1$ carried $= 1$, write down 1 $\therefore$ product is 134,757.

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[*] Rules derived from actual multiplication.

4. Write down the products in the following cases:—2357×31, 48765×41, 987643×51, 2131476 by 91, 28745 by 101.

5. Write down 4627143×71, 6219473 by 121, 81974×118, 7648×125, 82143×99, 267×1001.

6. Simplify $35^2 - 25^2$.

Sol.—$35^2 - 25^2 = (35 + 25)(35 - 25 = 60 \times 10 = 600$.

7. Simplify $85^2 - 65^2$, $95^2 - 75^2$, $65^2 - 35^2$ $312^2 - 311^2$, $629^2 - 621^2$.

8. Write down the results in the following cases:—7210^2, 7207^2, $1376^2 - 1370^2$, $1321^2 - 1221^2$, 1619^2, $- 1609^2$, $12105^2 - 12005^2$, $(164369)^2 - (164119)^2$.

9. Find the price of 288 yards of linen at $9\frac{3}{4}d.$ per yard.

Sol.—288 at $9\frac{3}{4} = 72$ at 3*s.* 3*d.* $= 9$ at 26*s.* $=$ £11 16*s.*

10. Find the cost of 512 yards of cotton at $6\frac{1}{4}$ cents.

Sol.—512 at $6\frac{1}{4} = 128$ at $25 = 32$ at \$1 $=$ \$32.

11. Find cost of 280 yards at $6\frac{3}{4}$ cents; 376 yards at $7\frac{1}{2}$ cents.

12. Find the cost of 960 yards of dress goods at 1*s.* $3\frac{3}{4}d.$

13. Find the cost of 1,216 yards at $3\frac{3}{4}$*d* ; 72 yards at 4*s*. $4\frac{1}{4}$*d*.

14. $37\frac{1}{2}$ lbs. of tea at 40 cents ; 96 lbs. at 3*s*. $4\frac{1}{2}$*d*.

15. Find the cost of 236 yards of Canadian tweed at 80 cents a yard.

16. Bought 24 yards of silk at 7*s*. 6*d*. sterling a yard : find the cost in Canadian currency, the shilling sterling being worth $24\frac{1}{3}$ cents.

17. Cost in Canadian currency of 360 yards of cotton at $6\frac{1}{4}$*d*. stg. a yard.

18. Find the cost in Canadian currency of 128 yards of dress goods at 1*s*. 9*d*. stg. a yard.

Section VI.—Mensuration.

Examples.—25.

1. A board is $15\frac{1}{2}$ inches wide and 16 feet long : how many square feet does it contain?

Sol.—Area = length × breadth $= 1\frac{7}{24}$ feet × $16 = \frac{31}{24} \times 16 = \frac{31}{3} \times 2 = 20\frac{2}{3}$ sq. feet.

Or: Calculate first for 12 feet long; then a board $15\frac{1}{2}$ inches wide and 12 feet long = one 12 in. wide and $15\frac{1}{2}$ ft. long $= 15\frac{1}{2}$ ft. Then 12 ft. long gives $15\frac{1}{2}$ ft. arc ∴ 16 ft. long gives $\frac{1}{3}$ more, $= 15\frac{1}{2} + \frac{1}{3}$ of $15\frac{1}{2} = 15\frac{1}{2} + 5\frac{1}{6} = 20\frac{2}{3}$.

2. Find area of a board 15 feet long, 13 inches wide; 18 feet long, 14 inches wide; 14 feet long, $16\frac{1}{2}$ inches wide.

3. How much lumber, inch measure, in 6 planks, each $18\frac{1}{2}$ inches wide, 16 feet long, and 3 inches thick?

> *Sol.*—$18\frac{1}{2}$ in. wide, 16 ft. long $= 24\frac{2}{3}$ (*i.e.*, $18\frac{1}{2} + \frac{1}{3}$ of $18\frac{1}{2}$) long, and 12 in. (wide) $= 24\frac{2}{3}$ ft. This × by $3 = 74$ ft. = amt. in 1 plank $74 \times 6 = 644$ ft.

4. How much lumber in 12 deals 18 feet long, $15\frac{1}{2}$ inches wide, and 3 inches thick?

5. How many square yards in the floor of a room 17 feet long and 16 feet wide, and what will it cost to carpet it at $1.80 per square yard?

> *Sol.*—Dimensions are $5\frac{2}{3}$ yds., $5\frac{1}{3}$ yds.; are $= 5\frac{2}{3} \times 5\frac{1}{3} = 25 + 5 + \frac{2}{9} = 30\frac{2}{9}$ yds., which, at $1.80, = $54.40.

6. What is the area of the floor of a room 16 ft. 6 in. square, and what will it cost to carpet it with carpet 1 yd. wide and $1.20 a yard?

7. If the carpet is only 27 inches wide, what will be the cost in last question?

> *Sol.*—No. sq. yds. in room $= 30\frac{1}{4}$, and cost = $36.30 when carpet is 1 yd. wide; but if carpet is 1-*fourth less* in width, the cost will be 1-*third* more = $36.30 + $12.10 = $48.40

8. What will it cost to carpet a room 23 ft. long and 22 ft. 6 in. wide, with carpet 27 in. wide, and costing $1.35 per yard?

9. Hall is 29 ft. 6 in. by 11 ft. 3 in.; carpet $\frac{5}{8}$ yd. wide, and 90 cents a yard: find cost.

> *Sol.*—$\frac{59}{6} \times \frac{15}{4} \times \frac{8}{5} = 59$ yds., which, at 90 cents = $53.10.

10. A hall is 10 yards long and 3 wide, what will it cost to carpet it with carpet 27 inches wide, at \$1.75 a yard?

11. If, in last question, the carpet is 45 inches wide, and same price per yard, what will [illegible] cost be?

Sol.—$10 \times 3 = 30$ sq. yds.; if carpet was 1 yard wide, the cost would be $30 \times 1\frac{3}{4} =$ \$$52\frac{1}{2}$, but it is 1-*fourth* more ∴ the quantity (and cost) will be 1-*fifth* less, and \$$52\frac{1}{2}$ — $\frac{1}{5}$ of \$$52\frac{1}{2}$ = \$42.

12. It costs \$40 to carpet a room which is 18 feet long and 15 feet wide: what would have been the cost if it had been 18 feet wide?

Sol.—18 ft. is $\frac{1}{5}$ more than 15 ft. ∴ cost would be $\frac{1}{5}$ more = \$40 + $\frac{1}{5}$ of \$40 = \$48.

13. If in last example the room had been a yard longer and a yard wider, what would have been the cost?

Sol.—Square yds. = 30; $\frac{1}{6}$ longer would make $30 + 5 = 35$, and $\frac{1}{5}$ wider would make $35 + 7 = 42$ yds., which will cost $\frac{2}{5}$ more than 30 yds., or \$56.

14. What would have been the cost in last example, if the room had been a yard longer, and a yard less in breadth? If it had been a yard shorter as well as a yard narrower?

15. The length of a room is $\frac{1}{5}$ more than its breadth; it takes 30 square yards to carpet it: find the length and breadth.

Sol.—Length = $\frac{6}{5}$ of breadth ∴ length × breadth = $\frac{6}{5}$ of square of breadth = 30 yds., and square of breadth = 25 yds. ∴ breadth = 5 yds., length 6 yards.

16. The breadth of a hall is $\frac{3}{8}$ of its length, and it costs \$25.60 to cover it with English oil-cloth, at \$1.60 a yard: find the length and breadth.

EXAMPLES.—26.

1. Find the cost of papering the walls of a room 16 ft. 6 in. long, 13 ft. 6 in. wide, and 10 ft. 6 in. high, the paper being 27 inches wide, and 20 cts. a yard.

> *Sol.*—$2 \times 5\frac{1}{2} + 2 \times 4\frac{1}{2} = 20$ (yds.) = circuit of room, then $3\frac{1}{2} \times 20 = 70$ yds.; if paper were a yard wide it would cost $70 \times 20 =$ \$14.00, but it is 1-*fourth* less than a yard wide ∴ cost will be 1-*third* more; *i.e.* \$$4\frac{2}{3}$ more ∴ cost = \$14 + \$$4\frac{2}{3}$, = \$$18\frac{2}{3}$.

2. A room is 18 ft. 9 in. long, 12 ft. 9 in. wide, and 10 ft. high: what will it cost to paint its walls at 22 cts. a yard, allowing 6 square yards off for the evenings?

3. It costs \$28 to paper the walls of a room 15 ft. square, with paper 27 inches wide, and 30 cents a yard: find the height of the room.

4. A school room is 30 ft. wide, and 36 feet long, and is seated for 60 pupils; how many square feet are allowed, on the average, to each pupil?

5. If, in the last example, the room is 12 feet high; how many cubic feet of space is allowed each pupil?

6. A rectangular boiler is 2 ft. 2 in. long, 1 ft. 4 in. wide, and $9\frac{1}{4}$ in. deep: how many gallons will it hold if a pint contains $34\frac{2}{3}$ cubic inches?

> *Sol.*—$26 \times 16 \times 9\frac{1}{4} \times \frac{3}{104} = 12 \times 9\frac{1}{4} = 111$ pints = $13\frac{7}{8}$ gal.

7. A plate of copper 1 ft. 4 in. long, 10 in. wide, and $\frac{3}{4}$ of an in. thick, is rolled into a sheet 2 ft. 6 in. long and 8 in. wide: find the thickness of the sheet.

8. A cistern is 4 ft. long, $3\frac{1}{2}$ ft. wide, and 4 ft. deep: what will it cost to cover the bottom and sides with lead, at 90 cts. per square yard?

9. What was the cost, at 90 cents per cubic yard, of digging a cistern 6 ft. long, 6 ft. wide, and 5 ft. 6 in. deep?

10. The length of a room is to its breadth as 4 : 3, its height is 10 feet, and it contains 1,080 cubic feet of air: find its length and breadth.

> *Sol.*—Cubic feet$=\frac{4}{3}$ of square of breadth $\times 10=1080$, and $\frac{4}{3}$ of square of breadth$=108$, ∴ square of breadth$=81$ ∴, breadth$=9$.

11. The length of a room which contains 45 cubic yards, is $\frac{1}{4}$ more than the breadth, the length being 9 feet: find the length and breadth.

12. Find the value, at $3.50 a cord, of a pile of wood 96 ft. long and 9 ft. high.

> *Sol.*—Pile is 12 cords long and $2\frac{1}{4}$ cords high; $2\frac{1}{4} \times 12=27$ cords—cost, $27 \times 3\frac{1}{2}=\$94.50$.

13. A pile of wood is 128 ft. long and 7 ft. high: find its worth at $4.25 per cord.

14. The length of a bath room whose ceiling is $10\frac{1}{2}$ ft. high, is $\frac{1}{2}$ more than the breadth, and the area of its walls is 35 yards: find the length and breadth.

> *Sol.*—Length$=1\frac{1}{2}$ breadth ∴ circuit of room $=2$ breadth $+$ 3 do.$=5$ times breadth, and this $\times 3\frac{1}{2} = 35$ yards ∴ 5 times breadth $= 10$ yards, and breadth$=2$ yards.

15. The length of a room is $\frac{1}{3}$ more than the breadth, the height is 9 ft. 9 inches, and the area of the walls 39 square yards : find the length and breadth.

16. A wall, whose height is 4 times its thickness, and whose length is 5 times its height, contains 2,160 cubic feet : find the dimensions of the wall.

17. Find the area of a circle whose diameter is 7 feet. *Note.*—Area of circle$=\frac{22}{7}\times$ square of radius.

Sol.— Radius$=\frac{7}{2}$ $\therefore$ area$=\frac{22}{7}\times\frac{49}{4}=\frac{77}{2}=38\frac{1}{2}$.

18. What is the area of a circle whose radius is 7 feet? 1 ft. 9 inches? $3\frac{1}{2}$ inches?

19. Out of a circular plate of 9 inches radius, is cut a concentric circular plate of radius 6 inches : find the area of the circular ring (annulus.)

Sol.—$\frac{22}{7}(9+6)(9-6)=\frac{22}{7}\times 45=141\frac{3}{7}$.

20. Find the area of a circular ring whose inner and outer diameters are 10 and 4 respectively.

21. Find the number of cubic inches in a cylindrical vessel whose inner diameter is 8 inches, and height 7 inches.

22. A hollow metallic column is 10 feet long, its diameter is 10 inches, and its thickness 3 inches : find the number of cubic inches of metal.

Sol.—Radii 5 in: and 2 in: $\therefore$ area of circular ring of section of column$=\frac{22}{7}(5+2)(5-2)=66$, and $\therefore$ $66\times 120=7920$ cubic inches.

23. An iron pipe is 14 feet long, its diameter from outside to outside is 8 inches, and the thickness is 2 inches : find the quantity of metal in the pipe.

24. Two circular plates of gold, each an inch thick, and of diameters 6 inches and 8 inches respectively, are melted into a single plate of same thickness: find its diameter.

> *Sol.*—Since thickness is same, the square of radius of new plate $=3^2+4^2=25$ ∴ radius $=5$, diameter 10.

25. Two circles have radii 8 feet and 6 feet respectively: find the radius of a single circle equal to both in area.

26. There is a circular annulus of gold plate 1 inch thick, whose outer and inner diameters are 10 inches and 4 inches respectively; it is melted into a single plate of gold whose diameter is 14 inches: find its thickness.

27. If the radius of the silver dollar be to that of the half dollar, as 3 : 2, compare their thicknesses.

> *Sol.*—Their values are proportional to their solid contents, let t, t, be the thickness, ∴ solid contents are proportional to $\frac{3^2\,t}{2^2\,t'}=\frac{2}{1}$ ∴ $\frac{t}{t'}=\frac{8}{9}$ the required ratio.

23. If the diameter of a sovereign be to that of a guinea as 7 : 8, find the ratio of their thicknesses.

CHAPTER III.

GENERAL ANALYSIS.

EXAMPLES 27.

1. A can do a piece of work in $\frac{1}{3}$ of a day and B in $\frac{1}{4}$ of a day: how long will it take both to do it?

> *Sol.*—A can do 3 times the work in 1 day, B

4 times ∴ both can do 7 times the work in 1 day, and the work in $\frac{1}{7}$ of a day.

2. Two pipes can fill a cistern in 3 hours; one of them can do it in 5 hours: how long will it take the other?

Sol.—Both can fill $\frac{1}{3}$ of cistern in 1 hour, one can fill $\frac{1}{5}$ ∴ the other can fill $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$ in 1 hour, and the whole in $7\frac{1}{2}$ hours.

3. If 2 men can reap a field in 6 hours, and one of them can do it in 10 hours, how long will it take the other?

4. A and B can mow a field in 2 hours, B and C in 4 hours, and A and C in 3 hours.

Sol.—A's work per hour + B's $= \frac{1}{2}$, B's + C's $= \frac{1}{4}$, C's + A's $= \frac{1}{3}$ ∴ twice (A's + B's + C's) $= \frac{1}{2} + \frac{1}{4} + \frac{1}{3} = \frac{13}{12}$, and A's + B's + C's $= \frac{13}{24}$, but A's + B's $= \frac{1}{2}$ ∴ C's $= \frac{13}{24} - \frac{1}{2} = \frac{1}{24}$ ∴ C in 24 days, &c.

5. A and B can dig a cellar in 6 days, B and C in 12 days, and A and C in 8 days; how long would it take each?

6. John can do a work in $\frac{3}{5}$ of a day and James in $\frac{2}{3}$ of a day: how long would it take them both together?

7. A can do a work in 2 days, B in 3 days, but with the aid of C they can do it in $\frac{6}{11}$ of a day: how long would it take C alone?

8. A and B do a work in 4 days; B can do in a day only $\frac{3}{5}$ as much as A: how long would it take each?

Sol.—B in 1 day does $\frac{3}{5}$ as much as A .·. in 4 days 4 × $\frac{3}{5} = \frac{12}{5}$ A's work in 1 day .·. A working for 4 days + $\frac{12}{5}$ days does the whole work.

9. A and B do a work in $\frac{7}{8}$ of a day, A does per day $\frac{1}{2}$ more than B: how long would it take each?

10. A does twice as much work as B in a given time, and C does $\frac{4}{3}$ as much work as A and B together, and all together can do a piece of work in $2\frac{4}{7}$ days: how long would it take each?

Sol.—A's work per day + B's + C's = 2 B's + B's + 4 B's = 7 B's work per day = $\frac{7}{18}$ of work .·. B in 18 days, &c.

11. A does $\frac{7}{8}$ as much work as B, and C does $\frac{2}{5}$ as much as A and B, and all do a job in 7 days: find the time for each.

12. Two men or 3 boys can do a work in $\frac{1}{6}$ of a day: how long will it take 3 men and 2 boys to do it?

13. To do a piece of work A requires twice as long as B and C, and C 3 times as long as A and B together; they all together do a piece of work worth $120: how should the money be divided?

Sol.—B's work per day + C's = 2 A's .·. A's + B's + C's = 3 A's .·. A does $\frac{1}{3}$ of work and is entitled to $40. So, A's + B's = 3 C's .·. A's + B's + C's = 4 C's, C does $\frac{1}{4}$ &c.

14. D can dig a ditch in 9 days, and D and E in 6 days: how long will it take E to do what remains after D has done $\frac{2}{3}$ of it?

15. C can do $\frac{2}{9}$ as much work as A and B together, and B as much as A and C together; they all together do a piece of work worth \$110: how much should each receive?

EXAMPLES.—28.

1. At what time after 3 o'clock are the hour and minute hands of a clock together?

> *Sol.*—Minute hand gains 11 rounds in 12 hours; at 3 o'clock, hour hand is $\frac{1}{4}$ round in advance; 1 round in $\frac{12}{11}$ hour, $\therefore$ $\frac{1}{4}$ round in $\frac{1}{4} \times \frac{2}{11} = \frac{3}{11}$ hour past 3 o'clock.

2. What time after 15 minutes past 7 will the hands of a clock be together?

3. At $\frac{1}{2}$ past 4 o'clock how many minutes will elapse before the minute hand will be opposite the hour hand?

4. What is the first time after 8 o'clock that the hands of a watch will be five minutes' space apart?

5. At a $\frac{1}{4}$ to 4 o'clock how many minutes will elapse before the minute hand is three hours' space in advance of the hour hand?

6. What is the first time after twelve o'clock that the hands of a clock will form an angle of 120 degrees?

7. The time since 6 o'clock is $\frac{5}{7}$ of the time to noon: what o'clock is it?

> *Sol.*—Time to noon $+ \frac{5}{7}$ of do. $= \frac{12}{7}$ of do. $= 6$ hours $\therefore$ $\frac{1}{7}$ of do. $= \frac{1}{2}$ an hour, and the whole $= 7 \times \frac{1}{2} = 3\frac{1}{2}$ hours $\therefore$ $8\frac{1}{2}$ hours a.m.

8. If $\frac{1}{2}$ the time past noon is equal $\frac{1}{4}$ of the time to midnight, find the time.

9. Half the time since midnight is ⅓ of the time to 10 o'clock a.m.: what o'clock is it?

10. A clock which is right on Monday at noon, loses 2 minutes in 4 hours: what is the true time when it shows 4 o'clock p.m. on Tuesday?

11. What is the hour if ⅔ of the time since 10 o'clock a.m., is the time to 3 o'clock p.m.?

12. Supposing the minute hand of a clock to move from right to left, while the hour hand moves in the ordinary way, and that the hands are together at 12, what hour will they indicate when they are next together.

13. Find the time when 1⅔ of time since 9 o'clock a.m. is the time to 9 o'clock p.m.

14. It is between 9 and 10 o'clock, a line joining the points of the minute and the hour hand forms an isosceles triangle having each of the angles at the base double of the third angle: find the time.

15. What is the hour when ½ the time since 6 p.m. is $\frac{1}{11}$ of the time since midnight?

16. Two watches hang side by side, and both show 12 o'clock at the time of observation; one keeps correct time, the other loses 10 minutes in 12 hours: in how many hours will the minute hands be at right angles to each other?

17. There is a mechanical contrivance resembling a clock; numbers from 1 to 20 are marked at equal intervals round the dial; the hands move in the same direction, one passing over two spaces in ¾ of a minute, while the other passes over 10. If the hands are placed together at 8, what number will they indicate when they are next together?

EXAMPLES.—29.

1. The sum of 2 numbers is 40, their difference is 8 : find the numbers.

> *Sol.*—The less + the less + 8 = 2 times less + 8 = 40 ∴ less 16, greater 24.
>
> *Note.*—The less = $\frac{1}{2}$ sum — $\frac{1}{2}$ the difference ; the greater = $\frac{1}{2}$ the sum + $\frac{1}{2}$ the difference.

2. The sum of two numbers is 25, their difference is 8 : find the numbers.

3. The sum of two fractions is $\frac{7}{9}$, their difference is $\frac{1}{3}$: find them.

4. The sum of two mixed numbers is $8\frac{5}{6}$, their difference is $2\frac{1}{2}$: find them.

5. If a man can row 8 miles per hour on still water, and a stream flows at the rate of 4 miles an hour, find the sum of their rates per hour.

6. If a man can row 7 miles per hour on still water, and a stream flows at the rate of 2 miles per hour, find the difference of their rates per hour.

7. If a man can row at the rate of 7 miles per hour on still water, and starts down a stream flowing at the rate of 3 miles per hour, how far down the stream will he go in 1 hour?

8. If a man can row at the rate of 7 miles per hour on still water, and starts up a stream which flows at the rate of 3 miles per hour, how far up the stream can he go in 1 hour?

9. If the sum of a man's and a stream's rate is 10 miles per hour, and their difference is 4 miles per hour, find their rates.

10. If, with the help of a stream, a man can go 10 miles down in 1 hour, and when the stream is against him he can come up 4 miles per hour, find their rates per hour.

11. A man, with the help of a stream, can row 24 miles in 4 hours: find the sum of their rates per hour.

12. A man rowing against a stream goes from A to B, a distance of 33 miles, in 6 hours: find the difference of their rates per hour.

13. A man can row 8 miles down a stream in 1 hour and 20 minutes, and he can row back again in 2 hours: find his rate per hour and the rate of the stream. *Ans.* 5 and 1 miles.

14. If a man can row down a stream 16 miles at the rate of 8 miles per hour, and back again at the rate of 2 miles per hour, compare the ratio his time coming up the stream bears to his time going down the stream, with the ratio the sum of their rates per hour bears to the difference of their rates per hour.

15. A man can row down a stream in 50 minutes, and back again in 70 minutes: compare his rate with the rate of the stream.

16. A crew can row up a stream a certain distance in 64 minutes, and back again in 60 minutes: determine the distance, the rate of the stream being half a mile per hour.

Sol.—Whole distance down in 1 hour, $\frac{15}{16}$ of distance up in 1 hour $\therefore$ $\frac{1}{16}$ of distance $=$ 2 rate of stream $=$ 1 mile, and distance $=$ 16 miles.

17. A man rows down a stream and back again; his rate is twice that of the stream: compare his rate going down with his rate coming up again.

18. A man rows down a stream and back again, his rate is 3 times that of the stream: compare the time going down the stream with the time coming up again; and what part of the whole time was spent going down stream.

19. A man can row down stream a certain distance and back again in 3 hours, and the rate of the stream is half his rate: how long is he going down, and how long coming up?

> *Sol.*—Rate *down* = rate of stream + 2 rate of do. = 3 times rate of do. Rate *up* = 2 rate of stream — rate of do. = 1 rate of stream ∴ rates are 3 : 1 : divide 3 hours in this ratio. 45 minutes, 2 hours 15 minutes.

20. A man can row 6 miles down a stream and up again in 2 hours 40 minutes, and his rate of rowing in still water is twice as great as the rate of the stream: find his rate of rowing.

21. How far may a boat whose velocity is 8 miles an hour in still water, go up a stream whose rate is 4 miles an hour, so that the round trip may take only 8 hours?

Examples.—30.

1. A merchant sold cloth for $2 a yard, and lost $6; had he sold it at $4 a yard he would have gained $18: how many yards did he sell?

Sol.—\$2 more on a yard makes a total difference of $\$6 + \$18 = \$24 \therefore 24 \div 2 = 12$ yards.

2. A boy bought some peaches for 3 cents each; had he paid 5 cents each, they would have cost 40 cents more: how many did he buy?

3. A train moves from A to B in 6 hours; if it increases its speed 10 miles per hour it will make the distance in 4 hours: find the distance.

4. A and B walk towards each other, A at the rate of 3 miles an hour, and B at the rate of 4½ miles an hour; when they meet it is found that B has walked 6 miles more than A: find their distance apart when they set out.

Sol.—In a dist. of $4\frac{1}{2} + 3 = 7\frac{1}{2}$ miles A makes $1\frac{1}{2}$ more than B $\therefore$ he makes 6 miles more in 30 miles.

5. A news-boy bought a number of papers at 2½ cents each, and had 5 cents left; had he paid 3 cents each, he would have been 7 cents short: find the number of papers.

6. A man's board was 75 cents a day, and his wages \$2.25; at the end of 40 days he had \$45: how many days was he idle?

7. A man agrees to dig a well for \$4 a day, on condition that he should forfeit \$5 for every idle day; he finished the well in 12 days, and received \$30: how many days was he idle?

Sol.—Had he worked all the 12 days he would have received \$48; but he received only

$30 .·. $48 — $30 = $18 loss; he loses $4 + $5 = $9 every idle day .·. $18 ÷ 9 = 2 idle days.

8. A man is to receive $1.50 for every day he works, and pay for every idle day 50 cents for board; at the end of 24 days he received only $24: how many days did he work?

9. A man received $2.50 a day for every day he worked, and paid 50 cents for every idle day; at the expiration of 40 days he had saved $50: how many days was he idle?

10. A man bought a number of sheep for $100; having killed 8 of them, he sold $\frac{2}{3}$ of the remainder at cost, receiving for them $40: how many did he buy?

Sol.—$\frac{2}{3}$ of rem. cost $40 .·. rem. cost $60, and the 8 killed must .·. have cost $100 — $60 = $40, or $5 each. 100 ÷ 5 = 20, number bought.

11. A man bought a number of calves for $80; he first sold 5 of them, and then $\frac{2}{3}$ of the remainder at cost for $40: how many did he buy?

12. A person bought a quantity of Canadian tweed for $150; after cutting off 6 yards, he sold $\frac{2}{3}$ of the remainder for $60: how many yards did he purchase?

13. A farmer bought a number of sheep, and having killed 10 of them, he sold, at cost, $\frac{7}{10}$ of the remainder for $294, which was $186 less than the entire lot cost: how many did he buy?

14. A dog killed $\frac{1}{4}$ of A's lambs; if he sells the remainder at cost he will receive $120; but reserving 8, and selling $\frac{1}{2}$ of the remainder at cost, he will receive $44: how many had he at first?

15. When John was 8 years younger his age was $\frac{3}{5}$ of his present age : how old is he?

EXAMPLES.—31.

1. If 20 lbs. of sea-water contain 1 lb. of salt, how much fresh water must be added so that 6 lbs. of the new mixture may contain $\frac{1}{5}$ lb of salt?

Sol.—$\frac{1}{5}$ lb. salt in every 6 lbs. of new mixture
$\therefore$ 1 lb. salt in every 30 lbs. of new mixture
30 — 20 = 10 lbs. fresh water added.

2. If 50 lbs. of sea-water contain 2 lbs. of salt, how much fresh water must be added so that 5 lbs. of new mixture may contain $\frac{1}{6}$ lb. of salt?

3. In a 60-ounce mixture of silver and copper, there are 4 oz. of copper : how much silver must be added to the mixture so that there may be $\frac{1}{9}$ oz. of copper in 2 oz. of the mixture?

4. If a sample of alcohol contains 80 parts of every hundred pure alcohol, and the rest water, how much water must be added so that the new mixture shall contain 60 parts in every hundred pure alcohol?

Sol.—60 parts alcohol in 100 of new mixture ; but there are 80 parts in all $\therefore$ if 60 parts be found in 100, 80 (which = 60 + $\frac{1}{3}$ of 60) will be found in 100 + $\frac{1}{3}$ of 100 = 133$\frac{1}{3}$
$\therefore$ 33$\frac{1}{3}$ parts are to be added.

5. How much water added to alcohol 96 per cent. strong will reduce it to 80 per cent. strong?

6. In a drove of 100 animals consisting of horses and cows, the latter are to the former as 2 : 3 : how many horses must be sold that there may remain 4 cows to 5 horses?

7. In a mixture of 120 lbs. of green tea and black, the quantity of green is to that of black as 3 : 5 : how much green tea must be added so as to make the ratio of green to black as 5 : 3 ?

8. How much water added to 25 gallons of alcohol 90 per cent. strong, will make it 75 per cent. strong ?

9. If 62 lbs. of sea-water contain 2 lbs. of salt, how much salt must be added so that 42 lbs. of sea-water will contain 2 lbs. of salt ?

> *Sol.*—In given mixture there are 60 lbs. fresh + 2 of salt; in new mixture there are 40 lbs. of fresh + 2 of salt. Then, if in 40 of fresh there are 2 of salt, in 60 of fresh there are 3 of salt ∴ 1 lb. of salt is to be added.

10. If 75 lbs. of sea-water contain 3 lbs. of salt, how much salt must be added so that 10 lbs. of new mixture will contain 1 lb. of salt ?

11. How much (weight) of fresh water must be added to 50 lbs. of brine 4 per cent. strong, to make the mixture 3 per cent. strong ?

12. How much alloy added to 9 oz. of gold 22 carats fine will make 18 carats fine ?

> *Sol.*—$\frac{22}{24}$ of 9 oz. $= 8\frac{1}{4}$ oz. of pure gold and $\frac{3}{4}$ of alloy in 1st case, in 2nd case the ratio is $18 : 6 = 3 : 1$, *i.e.*, the pure gold is 3 times the alloy which $\therefore = 8\frac{1}{4} \div 3 = \frac{11}{4} = 2\frac{3}{4}$ $\therefore 2\frac{3}{4} - \frac{3}{4} = 2$ oz. alloy to be added.

13. How much alloy added to 6 oz. of gold 20 carats fine, will give a sample 15 carats fine ?

14. In a mixture of gold and silver consisting of 50 oz. there are three oz. of silver: how much gold must be added that there may be $\frac{1}{20}$ oz. of silver to 1 oz. of gold?

15. A chemist dissolved 3 oz. of a salt in 27 oz. of water; but when his assistant by mistake added more water, he, to find the quantity of water added, evaporated 1 oz. of the new mixture and found $\frac{1}{15}$ oz. of the salt: how much water had been added by the assistant?

16. If 5 dwt. of gold 22 carats fine be mixed with 3 dwt. 14 carats fine, ascertain the fineness of the mixture.

Sol.—Fine gold in 1st $= 5 \times \frac{22}{24} = \frac{110}{24}$
" 2nd $= 3 \times \frac{04}{24} = \frac{42}{24}$
$\therefore$ " in 8 dwt. $= \frac{110}{24} + \frac{42}{24} = \frac{152}{24}$
and in 1 dwt. $= \frac{19}{24}$ $\therefore$ 19 carats fine.

17. If 8 oz. of gold 18 carats fine be melted with 4 oz. 15 carats fine, what is the fineness of the mixture?

18. A has $40 in gold and silver; for every $3 in gold he has $2 in silver: how much gold must be added so that there may be $3 of silver to $9 of gold?

19. 4½ dwt. of gold 18 carats fine are melted with 3½ dwt. 22 carats fine: how fine is the mixture?

20. If one gallon of alcohol 96 per cent. strong is mixed with one quart 81 per cent. strong, how strong is the mixture?

21. How much alcohol 90 per cent. strong must be mixed with 4 gallons 75 per cent. strong, to make the mixture 85 per cent. strong?

> *Sol.*—1 gallon of 90 per cent. is 5 per cent. *stronger* than gallon of new mixture; 1 gallon of 75 per cent. is 10 per cent. *weaker* than gallon of new mixture ∴ $\frac{1}{2}$ gallon of 75 per cent. compensates for 1 gallon of 95 per cent., and 4 gallons of 75 per cent. compensates for 8 gallons of 95 per cent., 8 is ∴ the number required.

22. How much alcohol 96 per cent. strong must be mixed with 8 gallons 60 per cent. strong to make mixture 80 per cent. strong?

23. How many dwt. of gold 15 carats fine must be melted with 15 dwt. 22 carats fine, to make a mixture 18 carats fine?

EXAMPLES.—32.

1. Charles is 16 years old and Willie is 7: in how many years will Charles be twice as old as Willie?

> *Sol.*—$16 - 7 = 9 =$ difference of ages, *which is constant.* At the required time C's age = 2 times W's, ∴ difference of ages is 2 times W's − W's = W's = 9, and $9 - 7 = 2$, the required time.

2. John is 28 years old and Mary 8: in how many years will John be 3 times as old as Mary?

3. A man is 40 years old and his son 20 years: how long since the father was 5 times as old as the son?

4. Charles is 18 years of age and Harry 15 years: how long since Harry was half the age of Charles.

5. Mary is 25 years old and Susie 7 years: in how many years will Susie be $\frac{1}{2}$ as old as Mary? $\frac{1}{3}$ as old?

6. A man is 48 years old and his son is 16 years: what was the man's age when he was 9 times as old as his son?

7. Julia's age is to Mary's as 3 to 2, and the sum of their ages is 25 years: in how long will Mary's age be to Julia's as 5 to 6, and what will then be the sum of their ages?

8. Two-thirds of John's age equals $\frac{4}{5}$ of William's, and the difference between their ages is 10 years: how long since John was 3 times as old as William?

Sol.—$\frac{2}{3}$ of J's = $\frac{4}{5}$ of W's age, and J's = $\frac{6}{5}$ of W's, and $\frac{6}{5}$ of W's — W's = $\frac{1}{5}$ W's = 10 $\therefore$ W's age = 50, J's = 60. Also, when W. was born J. was 10 years old, when will J. be 3 times as old? 3 W's age — J's age = 10 years $\therefore$ W's age = 5 years, which was 50 — 5 = 45 years ago.

9. Half of A's age equals $\frac{1}{3}$ of B's age, and the difference of their ages is 10 years; in how many years will $\frac{1}{3}$ of A's age equal $\frac{1}{4}$ of B's.

10. Two-thirds of A's age equals $\frac{3}{4}$ of B's age, and the sum of their ages is 68 years: how long since $\frac{2}{3}$ of A's age equalled $\frac{4}{5}$ of B's?

11. Mary is $\frac{1}{4}$ as old as her aunt, but in 20 years she will be half as old: find the age of each.

12. A man is twice as old as his wife; 16 years ago he was 3 times as old: find the age of each.

Sol.—Difference of ages=2 wife's age—wife's age=wife's age (present); also, difference of ages=3 wife's age—wife's age=2 wife's age (former)=wife's present age $\therefore$ 16 years = wife's former age.

13. John is 5 times as old as James, but in 8 years he will be only 3 times as old: find the age of each.

14. When A married he was three times as old as his wife, but in 15 years he was only twice as old: find the age of each at marriage.

15. Ten years ago A was four times as old as B, now he is only twice as old: find their ages.

16. Two trains moving in the same direction at rate of 10 miles an hour, pass a station, one 54 minutes behind the other: how many minutes will elapse before the forward train is 4 times as far from the station as the other?

EXAMPLES.—33.

1. How far may a person go in a stage which makes 6 miles an hour, so that by walking back at 4 miles an hour he may be gone only 5 hours?

> *Sol.*—He goes 1 mile in $\frac{1}{6}$ hour; he returns 1 mile in $\frac{1}{4}$ hour $\therefore$ he goes *and* returns 1 mile in $\frac{1}{6}+\frac{1}{4}=\frac{5}{12}$; and in 5 hours he goes and returns $5\div\frac{5}{12}=12$ miles.

2. How far may a person ride in a coach at the rate of 10 miles an hour, so that walking back at 5 miles an hour he may be gone only 6 hours?

3. How far may I sail in one steamer going at rate of 15 miles an hour, so that returning at rate of 12 miles an hour I may be gone but 9 hours?

4. A steamer whose rate of sailing is 12 miles an hour, descended a river whose current is 4 miles an hour, and returned; she was gone 6 hours: how far did she go?

5. A steamer whose speed is $10\frac{1}{2}$ miles an hour, plies between two cities on a river whose velocity averages $3\frac{1}{2}$ miles an hour: if the trip down and up takes 9 hours, what is the distance between the places?

Sol.—Rate down$=10\frac{1}{2}+3\frac{1}{2}=14$; rate up$=10\frac{1}{2}-3\frac{1}{2}=7$; then divide 9 hours in ratio of $14:7=2:1$ ∴ 3 hours for time down, and 6 hours for time up $=42$ miles (see on Ques. 1).

6. A boat which can make 12 miles an hour, takes 12 hours for the round trip on a river whose current is 4 miles an hour: how far does she go?

7. If a boat whose speed is 15 miles an hour is 3 hours making a trip down stream, and 6 hours in returning, what is the rate of the stream and the length of the trip?

8. I went to Niagara (from Toronto), in the "City of Toronto," which makes 14 miles an hour, and returned in the "Rothesay," which makes 15 miles an hour, and the round trip took 4 hours and 50 minutes: find the distance between Toronto and Niagara.

9. Six men hire a conveyance for a certain sum, but taking in two more, the expense to each was reduced by 50 cents: what was paid for the conveyance?

Sol.—Expense to 1 is diminished 50 cents ∴ expense to 6 is diminished $6\times\frac{1}{2}=\$3$ which the *two* pay, then 1 pays $\$1\frac{1}{2}$, and $8\times1\frac{1}{2}=\$12$ whole cost.

10. Ten men hire a coach for a certain sum, but taking in 5 persons more the expense of each is reduced 20 cents: what did the coach cost them?

11. Five men hire a conveyance, but 3 more join them, and the expense of each was reduced \$1.50: what was paid for the conveyance?

Sol.—3 men make a difference of $\$5\times\$1.50=\$7.50$.·. 1 man makes a difference of \$2.50 and $8\times2\frac{1}{2}=\$20$.

12. Three persons rented a church pew, and by taking in two more, the expense of each was diminished \$3: what was the rent of the pew?

13. Ten men chartered a boat, but 2 of them failed to pay, and the expense of each of the others was increased \$1.50: what was charged for the boat?

14. John bought 25 oranges and had 15 given to him, by which the average cost was reduced $1\frac{1}{5}$ cent: what did the 25 cost?

15. A news-boy bought 20 papers, but losing 5 of them, the cost of each was increased $\frac{1}{2}$ a cent: what did he pay for the papers?

EXAMPLES.—34.

1. There is a fish weighing 72 lbs.; his head weighs twice as much as his tail, and his body weighs as much as his head and tail together: find the weight of each part.

Sol.—Head $=2\times$ weight of tail, body $=$ tail $+$ head $=3\times$ weight of tail .·. head $+$ body $+$ tail $=$ whole weight $=6\times$ weight of tail $=72$.·. &c.

2. A fish weighs 64 lbs.; the head weighs 3 times as much as the tail, and the body weighs as much as the head and tail together: find the weight of each part.

3. A watch and seal are worth $160, the cost of the seal is $8 less than 20 per cent. of the cost of the watch: find the cost of each.

4. The head of a fish is four feet long, the tail is as long as the head and half the body, and the body is as long as the head and tail: find the length of the fish?

> *Sol.*—Head = 4 feet, tail = 4 feet + $\frac{1}{2}$ body $\therefore$ body = $\frac{1}{2}$ body + 8 feet $\therefore$ body = 16 feet, and 32 = whole length.

5. The head of a fish is 3 feet long, the tail is as long as the head and half the body, and the body is as long as the head and tail: find the whole length of the fish.

6. The tail of a pike weighs 3 ounces, the head weighs as much as the tail and $\frac{1}{4}$ of the weight of the body, and the body weighs twice as much as the head and tail: find the weight of the fish.

7. A boy spent $15, and then earned $\frac{1}{3}$ as much as he had remaining, and then found he had $\frac{1}{2}$ as much as he had at first: how much had he at first?

> *Sol.*—Remainder (after spending $15) + $\frac{1}{3}$ of remainder = $\frac{4}{3}$ of remainder = $\frac{1}{2}$ he had at first $\therefore$ $\frac{3}{3}$ of remainder = $\frac{3}{8}$ what he had at first $\therefore$ $\frac{5}{8}$ what he had at first = $15, and whole amount = $24.

8. A man spent $22, and then earned $\frac{1}{4}$ of what he had left, and then had $\frac{1}{3}$ as much as he had at first: how much had he at first?

9. A boy paid out 40 cents, and afterwards received 75 per cent. of as much as he had remaining; he then had $\frac{1}{6}$ more than he had at first: how much had he at first?

10. If my age 4 years hence be diminished by $\frac{3}{8}$ of itself, the remainder will equal $\frac{5}{6}$ of my age 6 years ago: find my age.

11. A boy went to a store and spent 21 cents, and then borrowing $\frac{1}{5}$ of what he had remaining, had $\frac{1}{2}$ as much as he had at first: how much had he at first?

12. A, B, and C live together for a certain time; A and B pay the entire expense in the ratio of 2 : 3; C gives $25: how should this be divided between A and B?

13. Suppose that for every 4 cows a farmer has, he should plow an acre of land, and allow one acre of pasturage for every 2 cows: how many cows could he keep on 18 acres?

> *Sol.*—4 cows require 1 acre plowed land $\therefore$ 1 cow requires $\frac{1}{4}$ acre, 2 cows require 1 acre pasture $\therefore$ 1 cow requires $\frac{1}{2}$ acre $\therefore$ 1 cow requires $\frac{1}{4}$ acre $+$ $\frac{1}{2}$ acre $=$ $\frac{3}{4}$ acre $\therefore$ 18 acres $\div$ $\frac{3}{4}$ acre $= 24 =$ number cows.

14. A farmer has to plow 1 acre for every 3 cows kept, and keep an acre of pasturage for every 2 cows: how many cows can be kept on 30 acres?

15. A farmer keeps 36 cows on 30 acres of plowed and pasture-land; he plows one acre for every 3 cows: how many acres of pasture must he allow for 4 cows?

16. A farmer keeps 48 cows on 14 acres of plowed and pasture-lands; he reserved 1 acre of pasture for every 6 cows: how many acres will he plow for every 16 cows?

Sol.—6 cows for 1 acre pasture ∴ 48 cows for 8 acres; hence 6 acres plowed for 48, and ∴ 2 acres for 16 cows.

EXAMPLES.—35.

1. I sold goods at 10 per cent. gain, but if they had cost $100 more, I should have lost 10 per cent. by selling as I did: find the cost of the goods.

Sol.—$\frac{9}{10}$ of new cost $= \frac{11}{10}$ of actual cost.
∴ new cost $= \frac{11}{9}$ of actual cost.
∴ $\$100 = \frac{2}{9}$ of actual cost, which ∴ $= \$450$.

2. Goods were sold at 10 per cent. gain, but if they had cost $60 more, there would, at the same selling price, have been 10 per cent. loss: find the cost of the goods.

3. A merchant sold goods at a gain of 20 per cent.; if they had cost $100 more, the same selling price would have entailed a loss of 20 per cent.: what did the goods cost?

4. A merchant marked cloth to make a profit, as he supposed, of 20 per cent.; but the cloth had cost $3 a yard more than he had supposed, and he therefore lost 25 per cent.: what did the cloth cost per yard?

Sol.—$\frac{3}{4}$ actual cost $= \frac{6}{5}$ supposed cost.
∴ actual cost $= \frac{8}{5}$ supposed cost.
∴ $\$3 = \frac{3}{5}$ of supposed cost, which ∴ $=$ $5, and $\$5 + \$3 = \$8$, actual cost.

5. A merchant sold goods at 20 per cent. gain; but if they had cost $150 less, his gain at the same selling price would have been 30 per cent. more: find the cost of the goods.

6. I sold tea at a gain of 25 per cent.; if it had cost 10 cents a pound less, the same selling price would have realized 50 per cent. gain: find the cost of the tea per pound.

7. Sold tea at a loss of 20 per cent.; if it had cost 25 cents a pound less, the same selling price would have gained 20 per cent.: find the cost of the tea per pound.

8. If an article had cost me 20 per cent. less, the gain would have been 30 per cent. more: what was the gain per cent.?

Sol.—Taking 1-*fifth* off the cost price (leaving the same selling price) would give the same increase in the percentage as adding 1-*fourth* to the selling price; then $\frac{1}{4}$ of selling price = (by question) 30 per cent. of cost ∴ selling price = 120 per cent. of cost; and gain was 20 per cent.

Or, new cost is 80 per cent. of the old; to sell at old cost would ∴ give a profit of 25 per cent. on new cost and nothing on old; also, 1 per cent. on old cost represents $1\frac{1}{4}$ per cent. on new cost, *i.e.*, every 1 per cent. on old cost represents increase of $\frac{1}{4}$ per cent. on new cost ∴ to give 30 per cent. — 25 per cent. = 5 per cent. we should require $5 \div \frac{1}{4} = 20 =$ per cent. on old cost.

9. If an article had cost me 10 per cent. less, the same selling price would have brought me 12 per cent. more: what was the gain per cent?

10. If the cost had been 25 per cent. less, the gain, on same selling price, would have been 16 per cent. more: find the gain per cent.

11. If the cost had been 8 per cent. less, the gain would have been 10 per cent. more: find the gain per cent.

Sol.—8 per cent. = $\frac{2}{25}$; $\frac{2}{25}$ off cost gives same increase in the per cent. as $\frac{2}{25-2}$ = $\frac{2}{23}$ added to selling price ∴ $\frac{2}{23}$ of selling price = (by question) 10 per cent. of cost price; $\frac{1}{23}$ of do. = 5 per cent. of cost ∴ selling price = 115 per cent. of cost, or gain is 15 per cent.

NOTE.—It will be seen that $\frac{1}{5}$ *off* cost gives same as $\frac{1}{4}$ on selling price; $\frac{1}{3}$ off is same as $\frac{1}{6}$ on; $\frac{1}{n}$ off same as $\frac{1}{n-1}$ on, and generally $\frac{a}{b}$ *off* cost = $\frac{a}{a-b}$ on selling price.

12. If cost had been 16 per cent. less, gain would have been 20 per cent. more: find the gain per cent.

13. If cost had been 4 per cent. less, the gain would have been $4\frac{2}{3}$ per cent. more: find the gain per cent. at which the goods were sold.

14. If cost had been 20 per cent. less, gain would have been 28 per cent. more: find gain per cent.

15. If cost had been 20 per cent. less, the gain, per cent. at same selling price, would have been *doubled*: at what per cent. profit were the goods sold?

Sol.—20 per cent. = $\frac{1}{5}$; $\frac{1}{5}$ off cost = $\frac{1}{4}$ on selling price, thus $\frac{1}{4}$ selling price = gain per cent. $\therefore \frac{3}{4}$ of selling price = cost hence there is gain of 1 in 3 = $33\frac{1}{3}$ per cent.

Illustration.—Let 100 = old cost, then 80 = new cost, and $133\frac{1}{3}$ selling price; now 80 sells for $133\frac{1}{3}$ = $66\frac{2}{3}$ per cent. gain.

16. If the cost of certain goods had been 25 per cent. less, the gain per cent. (same selling price) would have been trebled: find the gain per cent.

17. If the cost of an article had been 30 per cent. less, the gain per cent. would have been $2\frac{1}{2}$ times as great: find the gain per cent.

18. A merchant marked his goods to make a certain rate of profit; he afterwards found that the goods had actually cost him 5 per cent. less, and that his gain per cent. was in consequence 6 per cent. more: for what rate of profit did he mark his goods?

19. If the cost of certain goods had been 12 per cent. *more*, the gain per cent. would have been 15 less: find the gain per cent.

Sol.—12 per cent. = $\frac{3}{25}$; $\frac{3}{25}$ *increase* in cost = $\frac{3}{25+3}$ = $\frac{3}{28}$ *decrease* in selling price = (by question) 15 per cent. of cost.

$\therefore \frac{1}{28}$ of selling price = 5 per cent. of cost, and selling price = 140 per cent. of cost, and $\therefore$ gain was 40 per cent.

20. If cost of certain goods had been 10 per cent. more, the gain per cent. would have been 12 less: find the gain per cent.

21. If the cost had been 20 per cent. more, the gain per cent. would have been 25 less : find the gain per cent.

NOTE.—If cost is $\frac{1}{2}$ more, gain will be $\frac{1}{3}$ of selling price less; cost $\frac{1}{3}$ more, gain $\frac{1}{4}$ less ; cost $\frac{1}{n}$ more, gain $\frac{1}{n+1}$ of selling price, *less*, &c. If cost $\frac{a}{b}$ more, selling price $\frac{a}{a+b}$ less.

22. If the cost of goods had been 20 per cent. more, the gain per cent. would have been only one-half what it actually was : find the gain per cent.

Sol.—$\frac{1}{5}$ on cost, same as $\frac{1}{6}$ off selling price; hence $\frac{1}{6}$ selling price = $\frac{1}{2}$ gain per cent.; and selling price = 3 gain per cent., and gain per cent. = $\frac{1}{3}$ selling price $\therefore$ $\frac{2}{3}$ of selling price = *cost*; there is $\therefore$ a gain of 1 on 2, or 50 per cent.

23. I bought goods, and mark them to make a certain rate per cent. of profit ; I afterwards found that the goods had cost me 33$\frac{1}{3}$ per cent. more than I had thought, and my gain was only $\frac{1}{3}$ as much per cent. : at what per cent. profit were the goods marked ?

24. Two merchants sell cloth at the same marked price, but one, having given 25 per cent. more for the cloth than the other, makes 30 per cent. less. : what per cent. on cost is the marked price ?

EXAMPLES.—36.

1. A store-keeper, on a certain day, first took in as much money as he began the day with, then paid out \$10, then took in as much as he had left, then paid out \$20, and had \$40 left : how much cash did he begin the day with ?

Sol.—He had \$40 at closing ∴ \$40 + \$20 = \$60; \$60 ÷ 2 = \$30 = amount left after first payment (\$10) ∴ \$10 + \$30 = \$40 and \$40 ÷ 2 = \$20 = amount with which he began the day.

NOTE.—In such questions it is generally easier to begin with *last result*, and work backwards.

2. A man began the day with taking in as much cash as he had, then paid out \$6, then took in as much as he had left, then paid out \$3, and had \$25 left: how much had he at first?

3. B took in twice as much as he had and then paid out \$1, then took in 3 times as much as he had left, then paid out \$17 and had \$70 left: how much had he at first?

4. C first took in half as much money as he had, then paid out \$3, then took in $\frac{1}{3}$ as much as he had left, then paid out \$2, and had \$10 left: how much had he at first?

5. A man paid out \$22, then took in $\frac{1}{4}$ as much as he had left, and then had $\frac{1}{3}$ as much as he had at first: how much had he at first?

6. John borrowed as much money as he had, then spent 8 cents, then borrowed as much as he had left, and spent 30 cents, and had 34 cents left: how much had he at first?

7. A man spent $\frac{1}{2}$ his money and \$$\frac{1}{2}$ more, then $\frac{1}{2}$ of what remained and \$$\frac{1}{2}$ more, and then had \$6.75 remaining: how much had he at first?

Sol.—\$$6\frac{3}{4} + \frac{1}{2}$ = \$$7\frac{1}{4}$ = $\frac{1}{2}$ remainder after

> first payment. ∴ first payment = 2 × $7\frac{1}{4}$ = \$$14\frac{1}{2}$, and \$$14\frac{1}{2}$ + \$$\frac{1}{2}$ = \$15 = $\frac{1}{2}$ of original sum, which ∴ = \$30.

8. A man spent $\frac{1}{2}$ his money and \$5 more, then $\frac{1}{2}$ of what remained and \$$\frac{1}{2}$ more, then $\frac{1}{2}$ of what remained and \$$\frac{1}{2}$ more, and then had \$3 left: how much had he at first?

9. D first took in $1\frac{1}{2}$ times as much cash as he had, then paid out \$5, then took in $2\frac{1}{3}$ times what he had left and paid out \$8, and then had \$24 left: how much had he at first?

10. A boy spent $\frac{1}{2}$ his money and \$$\frac{1}{2}$ more, then $\frac{1}{2}$ the remainder and \$$\frac{1}{2}$ more, and then had \$3 left: how much had he at first?

11. A lady bought a silver watch for \$30, also a gold watch and a chain, which together cost 4 times as much as the silver watch; the chain and silver watch cost twice as much as the gold watch: find the value of each.

> *Sol.*—Gold watch = $\frac{1}{2}$ chain + \$15; chain + gold watch, *i. e.*, chain + $\frac{1}{2}$ chain + \$15 = \$120 ∴ $\frac{3}{2}$ of chain = 120 — 15 = \$105; chain = \$70, etc.

12. A man had \$6 in silver, and also some copper and gold; the copper with the silver was twice the gold; the silver with the gold was 5 times the copper: how much of each kind had he?

CHAPTER IV.

QUESTIONS FOR EXAMINATION.

I.

1. Increase a certain number by $33\frac{1}{3}$ per cent., extract the cube root, then multiply by 8, and extract the square root of the product—the result is four: find the number.

2. Divide \$148 among A, B, C, so that B may have \$10 more than C, and A \$17 more than B.

3. A boat floats down stream a certain distance in 10 hours, and is rowed back by a man in 4 hours: find in terms of the distance the man's rate of rowing in still water.

4. A person marks his goods so that he may allow a discount of 5 per cent. and still make a profit of 20 per cent.: what should be the marked price of an article that cost 95 cents?

5. A newsboy spent 85 cents in papers, taking 3 papers at $3\frac{1}{3}$ cents each as often as 2 papers at $3\frac{1}{2}$ cents each; he sold all at 5 cents each: how much did he gain?

> *Sol.*—3 at $3\frac{1}{3}=10$, 2 at $3\frac{1}{2}=7$; 85 divided in ratio of 10 : 7 gives 50 cents and 35 cents $\therefore$ 15 papers and 10 papers, etc.

6. A note for \$150, drawn at 4 months, and bearing interest at 8 per cent., is discounted by a broker 2 months before maturity, the broker makes 12 per cent. (per annum) on his money: what does he pay for the note?

7. Find a fourth proportional to 108, 972, 313.

8. A man willed $2,200 to two children, whose ages were 11 years and 16 years respectively, in such a way that the shares would, at five per cent. simple interest, amount to equal sums when the children became of age: what were the shares?

9. If I retail goods at a profit of 20 per cent., and sell at wholesale for 25 per cent. less than at retail, what rate per cent. do I gain at wholesale?

10. A sum of money amounts to $370 in six years, and to $330 in four years: find the sum of money and the rate.

11. A has three times as much money as B, but if he gives B $8 they will have equal sums: how much has each?

II.

1. The circumference of the fore-wheel of a carriage is 12 feet, of the hind wheel 14 feet: how far will the carriage go before two points, at the same time in contact with the ground, will again touch the ground together?

2. Find the smallest sum of money that can be exactly paid with guineas, sovereigns, or marks.

3. If a number be increased 25 per cent., and the sum be increased by 20 per cent., the result will be 30: find the number.

4. A and B own 40 per cent. and 60 per cent. respectively of a dish of berries; C joins them, and the three eat the berries; C gives 30 cents for his share: how much of this should A get?

5. The interest on a sum of money for a certain time and rate is $110 ; the discount is $88 : find the sum.

> *Sol.*—$100—$88=$22, which is interest on discount ; $88 produces $22 interest, and $\frac{22}{88}=\frac{1}{4}$; *i.e.*, the interest$=\frac{1}{4}$ principal $\therefore 110\times 4=440$, the required **sum.**

6. A sold a horse to B for $200, and took his note payable in 70 days. A got the note discounted at the bank at 10 per cent., and found he had made a profit of $16\frac{2}{3}$ per cent. : what did the horse cost A ?

7. A father divided $48 among 2 sons and 3 daughters, giving each son half as much as each daughter : how much did each receive ?

8. Bought 80 barrels of flour, part at $10 and the rest at $8 a barrel ; the whole cost $740 : how many barrels of each kind were there ?

9. The difference between one number and $\frac{2}{3}$ of another is 7 ; the sum of the numbers is 32 : find them.

10. A fish can swim from A to B and back again in 4 hours, swimming against the stream 4 times as fast as the rate of the stream : find the rate of the stream in terms of the distance.

III.

1. In a cricket match the scores in each successive innings are $\frac{1}{3}$ less than in the preceding innings, and the side which has had the first innings wins by 20 : what are the scores in each innings ?

2. If a crew can row from A to B in 40 minutes and back again in 35 minutes, compare the rates of the stream and boat ?

3. John can run 10 yards whilst James runs 11 yards: what start ought James to give John in a quarter of a mile race so as to win by 1 yard?

4. James has 10 marbles, John has as many as James and $\frac{2}{3}$ of Robert's, and Robert has as many as John and James: how many have they all?

5. At what time between 6 and 7 will the hour and minute hands of a clock point directly opposite? At what time between 2 and 3? Between 7 and 8?

6. A man sold cider that cost $2.50 a gallon for $3. The price of the cider rose to $3, he watered it so that he could still sell it at $3 and make the same profit as before: how much water did he put with 10 gallons of cider?

7. A person insured his life for $6,000 at the rate of $6.20 per cent. After paying 4 premiums he died: how much more did his family receive than was paid out for premiums?

8. A and B agreed to cut some wood for $40; when the work was partly done A was taken sick and received only $10: what part of job was finished when A took sick?

9. How long will a boy take to walk around an oblong plot of ground $\frac{1}{4}$ of a mile long by $\frac{1}{8}$ of a mile wide, if he takes 15 steps of two feet each in half a minute?

10. Divide $2 between 3 persons, so that the first may have $\$\frac{1}{5}$ more than the third, and $\$\frac{7}{10}$ less than second.

IV.

1 Two-thirds of the square of twice a number is equal to $\frac{4}{3}$ of the square of $\frac{3}{2}$ of the number diminished by 3 : find the number.

2. The sum of two numbers is 40, the difference and $\frac{1}{3}$ of one of them is 20 : find the numbers.

> *Sol.*—The difference + $\frac{1}{3}$ of one of them = difference between one and [illegible] of the other, ∴ first=20+$\frac{2}{3}$ of second, and ∴ second+$\frac{2}{3}$ of do.+20=40; hence $\frac{5}{3}$ of second =20 ; second 12, first 28.

3. A quantity of tea was sold at 10 per cent. gain : if it had cost \$60 more the same selling price would have entailed 10 per cent. loss : find the cost of the tea.

4. A merchant laid out \$1,000 in buying cloth, paying 25 cents a yard for 25 per cent. of the cloth, and 33$\frac{1}{3}$ cents a yard for the remainder : how much will he clear by selling the cloth at 50 cents a yard?

5. A kind of brass is made by fusing together old brass, refined copper, and zinc, in the proportions of 3, 4, 5 : how much of each kind must be taken to produce 150 lbs. of brass, after allowing 6$\frac{1}{4}$ per cent. for waste?

> *Sol.*—6$\frac{1}{4}$ per cent.=$\frac{1}{16}$ of amount wasted ∴ $\frac{15}{16}$ of do.=150, $\frac{1}{16}$=10 and $\frac{16}{16}$=160 lbs. to be taken ; divide this in ratio of 3, 4, 5.

6. The numerator of a fraction is 25 per cent. less than the denominator, and the sum of both is 448 : find the fraction.

7. The dimensions of a writing-table which contains 2$\frac{4}{9}$ square yards, are as 9 : 11 : find them.

8. A, B, C trade in company ; A's capital is to B's as 3 : 5 ; C took $100 of the $420 gained during the half year : find the gains of A and B, also C's stock.

9. The base of a right-angled triangle is 60 per cent. of the hypotheneuse, the perimeter being 240 : find the area of the square described on the perpendicular from the right angle on the hypotheneuse.

10. If the cost had been 20 per cent. less, the loss would have been 15 per cent less : find loss per cent.

V.

1. How much carpeting $\frac{3}{7}$ of a yard wide will be required to cover a floor 15 feet long and 12 feet wide?

2. The quotient of a division is twice the remainder, and the division twice the quotient; the sum of eight times the divisor, increased by 8, and three times the remainder, decreased by 4, will equal 18 times the quotient : find the dividend.

3. A father divided among his three sons, A, B, and C, $4,000 of Grand Trunk Railway stock, so that the sum of A's share and half the united shares of B and C, will be $2,750, and the sum of B's share and half the united shares of A and C, will be $2,450 : how much stock must B obtain from A and C respectively, so that each son may have the same amount of stock ?

4. A and B agree to perform a job of work which they can do in 30 days, working 10 hours a day. If they complete the work before the 30 days have expired, they are to have $2 extra for every day intervening between the completion of the work and the expiration of the specified time. A can do as much work in 5 hours as B can do in 6; B works 2 hours extra per day for 15 days, and A $2\frac{1}{2}$ hours extra per day for 12 days: how should the money be divided?

5. A person lent a certain sum of money at 8 per cent. per annum, interest payable half-yearly. In receiving the interest for the first half-year, he unfortunately took a counterfeit bill of $20. The remainder of the half-year's interest was to the whole half-year's interest as 7 : 8: find the principal.

6. A certain sum is put out at interest at 10 per cent.; $3\frac{1}{2}$ times the interest for one year × $20 equals $\frac{4}{11}$ of the amount for one year — $15: required the principal?

7. There are two numbers such that $\frac{1}{2}$ of the greater equals $\frac{2}{3}$ of the less. If $\frac{1}{2}$ the greater be divided by $\frac{1}{3}$ of the less, and this quotient by increased by $\frac{1}{2}$ the the sum of the two numbers, the result will be 128: find the numbers.

8. A can do a piece of work in 12 days; B can do in the same time $33\frac{1}{3}$ per cent. more work than A, but he does less than C by $33\frac{1}{3}$ per cent. of C's work; B and C work together 2 days, and then leave: how long will it take A to finish?

9. The difference between the simple and compound interest of a certain sum of money for 2 years at 6 per cent. is $3.20: find the principal.

10. The length of a room is $\frac{14}{5}$ times the height, and the height is $\frac{5}{7}$ of the breadth; it requires $106\frac{2}{3}$ yards of paper 2 feet wide to paper the walls: how much will it cost to plaster the ceiling at 15 cents per square yard?

VI.

1. The sum of three fractions is 2; the numerators, when reduced to equivalent fractions having least com. denominator, are 4, 3, 5: find the fractions.

2. A man bought stock at 25 per cent. below par, and sold it at 20 per cent. above par; he gained $900: how much stock did he buy?

3. A person discounts a note due in 15 months (without interest), so as to make 10 per cent. on his money: what rate of interest did he exact on the face of the note?

> *Sol.*—10 per cent. for 12 months = $12\frac{1}{2}$ per cent. = $\frac{1}{8}$ for 15 months ∴ what he pays for note + $\frac{1}{8}$ of ditto = $\frac{9}{8}$ of ditto = face of note ∴ he pays $\frac{8}{9}$ of face of note, *i.e.*, takes off $\frac{1}{9}$ or $11\frac{1}{9}$ per cent.; this for 15 months = $8\frac{8}{9}$ per cent. per annum.

4. A pipe $\frac{1}{2}$ an inch in diameter will fill a cistern in 20 minutes, the cistern being empty: how long a time will be required to fill it when there is a discharge pipe of $\frac{1}{4}$ inch diameter?

5. If 1 gallon of alcohol 96 per cent. strong is mixed with 1 quart 81 per cent. strong, what per cent. strong is the mixture?

6. Four circular plates of gold, two of them being of 3 inch radii, and the others of 4 inch radii, and all 1 inch thick, are melted into a single circular plate $\frac{1}{2}$ inch thick: find its radius.

7. The amount of A's money for 2 years at 5 per cent. is $60 more than its interest for 9 years at 10 per cent.: how much has he?

8. Two partners, A and B, gain $300; A owns within $40 of $\frac{2}{3}$ of the entire stock, and B's gain is $120: required each man's stock.

> *Sol.*—If A owned $\frac{2}{3}$ his gain would be $200; but his gain is only $180 ∴ $200 — $180 = $20 = gain for $40 (which is the amount *less* than $\frac{2}{3}$) = ∴ $40 gains $20, $1 gains $\$\frac{1}{2}$ ∴ $600 to gain $300.

9. Paid 25 per cent. duty on a watch, and sold it at a loss of $33\frac{1}{3}$ per cent.; had it been sold for $40 more, there would have been a gain of $16\frac{2}{3}$ per cent.: find the price at which the watch was invoiced.

10. How far may one ride in a stage making 5 miles an hour, that he may walk back at 3 miles an hour, and be gone but 8 hours?

VII.

1. A man invested $\frac{1}{3}$ of his money in bank stock at 120, and paying 8 per cent. dividends, and the remainder in railway stock at 80, and paying 5 per cent. dividends: his income from both investments was $115: find the amount of each investment.

> *Sol.*—In first case income is $\frac{1}{15}$ of investment
> ∴ on $\frac{1}{3}$ it $= \frac{1}{3} \times \frac{1}{15} = \frac{1}{45}$.
> In second case income is $\frac{1}{16}$ of investment
> ∴ on $\frac{2}{3}$ it $= \frac{2}{3} \times \frac{1}{16} = \frac{1}{24}$.
> ∴ total income $= \frac{1}{45} + \frac{1}{24} = \frac{8}{360} + \frac{15}{360}$
> $= \frac{23}{360}$ of investment = $115.
> ∴ $\frac{1}{360}$ of income = $5, whole = $1,800
> = or $600, $1,200.

2. A and B gain \$240 ; A owns $\frac{3}{4}$ of the stock lacking \$10, and B's share of the gain is \$65 : find the whole stock, and the share of each ?

3. The amount of a sum of money at a certain rate per cent. for 2 years is \$290, and for 4 years at $\frac{1}{2}$ the rate per cent. is \$330 : find the sum and rates.

4. A ball 3 inches in diameter weighs 4 ounces : what would a ball 2 inches in diameter weigh if composed of material 50 per cent. heavier than that of the first ?

5. The difference between the interest and the discount on a sum of money for $1\frac{1}{2}$ years at 8 per cent. is \$18 : find the sum of money.

Sol.—8 per cent. for $1\frac{1}{2}$ years = 12 per cent. $= \frac{3}{25}$ $\therefore$ discount $= \frac{3}{28}$ and $\frac{3}{25} = \frac{3}{28} = \frac{9}{700} =$ \$18 $\therefore$ \$1,400.

6. $\frac{2}{3}$ of the time since $7\frac{1}{2}$ o'clock a.m. equals $\frac{1}{2}$ the time to $9\frac{1}{2}$ o'clock p.m.: what o'clock is it ?

7. I sold goods at double their cost, but if the cost had been \$15 more, the gain would have been only 20 per cent.: find the cost.

8. The breadth of a room is to its length as 5 : 6 ; it costs \$30 to carpet it with material 27 inches wide and \$1.25 a yard : find the length and breadth of the room.

9. A man had to give \$51 in bills for \$50 in gold : at what rate of premium was gold ?

10. If a town pays its tax collectors 5 per cent. and allows 5 per cent. for uncollected taxes, what should be the amount of assessment to realize \$36,100 ?

VIII.

1. A can walk 10 miles in $2\frac{1}{4}$ hours, B can walk 15 miles in $3\frac{1}{3}$ hours. They start to walk a match from Toronto to Whitby, a distance of 30 miles: which will arrive first, and by what amount of time will he win?

2. By selling vinegar at 1 cent per pint above cost, the gain is 50 per cent.: what per cent. is gained when it is sold at 6 cents a gallon above cost?

3. If the income tax is 5 per cent. what income nets, after paying the tax of \$1,900?

4. The L. C. M. of two numbers is 72 and the G. C. M. is 3, one of the numbers is 9: what is the other?

5. From Toronto to Guelph is 48 miles, and $\frac{5}{4}$ of this distance plus 3 miles is $\frac{1}{44}$ of 7 times the distance from Berlin to Montreal: find the distance from Berlin to Montreal.

6. A person walks from Toronto to Brampton at the rate of 4 miles an hour, and after resting $\frac{1}{2}$ an hour rides back at the rate of 6 miles an hour and he finds he has been absent $10\frac{1}{2}$ hours: find distance between places.

7. A lady bought some eggs at the rate of 3 for 5 cents and had 10 cents left. Had she given 25 cents a dozen she would have needed 5 cents more to pay for them: how many did she buy?

8. A person sold two houses for \$3,960 each, making 10 per cent. on one and losing 10 per cent. on the other: taking both sales into account, what was his gain or loss?

9. A publisher takes off $33\frac{1}{3}$ per cent. of his retail prices for a wholesale customer: what per cent. will the wholesale customer gain if he sells at the publisher's retail prices?

10. A father left some money to be divided between his three sons in the proportion of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$; the money was invested at 6 per cent., the yearly interest being $234: how much of the principal should each receive?

IX.

1. How many yards of carpeting $2\frac{1}{2}$ feet wide are required to cover a hall floor 7 feet wide and 15 feet long?

2. The circumference of the hind wheels of a perambulator is $7\frac{1}{2}$ feet and of the forewheels $3\frac{1}{3}$: over what distance will they pass before they are in the same relative positions as at starting? How many revolutions will each wheel have made?

3. How far can a boy ride in a carriage going at the rate of 9 miles an hour, provided he walks back at the rate of 6 miles an hour, and is gone only 10 hours?

4. A pickpocket robbed a gentleman of $\frac{4}{5}$ of his money; when he was arrested he had made away with $\frac{3}{4}$ of it, and had $50 left: How much had the gentleman at first?

5. John and James have $60, and 50 per cent. of John's money equals 60 per cent. of James': how much had each?

6. A and B can do a piece of work in 6 days; A and C in 8 days, and B and C in 9 days: how many days would each take working alone?

7. A gang of thieves stole 26 fowls, half of them getting 4 each, the rest 3 each, except two who obtained but 2 each: how many were there in the gang?

8. If \$4 is allowed as 12 months' discount off a bill for \$76, and at the same rate \$7 be allowed off a bill for \$91, for how long was the latter sum discounted?

> *Sol.*—Disct. $= \frac{4}{76} = \frac{1}{19}$ ∴ *int.* $= \frac{1}{18}$: so disct. $= \frac{7}{91} = \frac{1}{13}$ ∴ int. $= \frac{1}{12}$; and since *int.* is proportional to the time, we have required time $= \frac{18}{12}$ of 12 months $=$ 18 months.

9. What is the least sum of money with which a farmer can purchase lambs at \$2½, sheep at \$7½, and pigs at \$3⅓ each, and how many of each can he buy with this sum of money?

X.

1. One person expends \$6 in coal at \$8 a ton, and another \$8 in coal at \$10 a ton: what fraction of a ton has the one more than the other?

2. Fifty men have provisions to last 60 days at a certain rate of supply. Ten more men coming, and the daily supply being made ⅓ less than before, how long will the provisions last?

3. If a certain number is increased by 6 and then multiplied by 3, the product divided by 9 and the quotient diminished by 4, the remainder is 8: what is the number?

4. If I ask 20% profit on an article, but fall 10% on my asking price, what rate of profit do I make?

5. Divide ⅜ into two parts, so that one may be $\frac{3}{40}$ less than the other.

6. An agent received $3,150, from which he was to take his commission on the sum invested. The commission was at the rate of 5% on the sum invested: how many acres did the agent buy if the land cost $20 an acre?

7. How many acres in an oblong garden, 40 rods by 15 rods? How many feet of lumber required to fence it with a tight board fence, 6 feet high?

8. A train starts at the rate of 30 miles an hour and runs at that rate for 20 minutes. It then increased its speed to 50% of its former rate: how far did it run in all during the first hour?

9. The product of three numbers is 105, two of them are to each other as $\frac{1}{6}$ to $\frac{7}{48}$, and the third is $7\frac{1}{2}$: what are the numbers?

10. I sold $700 worth of goods so as to gain $12\frac{1}{2}$% of the *proceeds*, and another lot which cost $400, so as to gain 20% of the proceeds: find gain % on the whole?

XI.

1. A room 24 feet long, 18 feet wide and 10 feet high, has a base board 6 inches wide all round: how many rolls of paper $1\frac{1}{2}$ feet wide and 14 yards long will it take to paper it, allowing 3 rolls for the openings?

2. What will 24 packs of envlopes cost, each pack containing 25, if a thousand cost $5?

3. A dog overtook a fox after running half a mile. Four-fifths of the distance the fox ran, after the dog started, was 8 rods less than 6 times the start he had: how many rods start had the fox?

4. A does a piece of work in 5 days, B in 6 days, and C in 8 days; $60 is paid for the work: what should each receive when they work together?

5. John has $30 in gold and silver, and for every $6 of gold he has $4 of silver: how much gold must be added that there may be $9 of gold for every $3 of silver?

6. A man floats down a stream a certain distance in an hour and rows back in twenty minutes: compare his rate of rowing in still water with the rate of the stream.

7. A boy having 60 pigeons in one coop and a smaller number in another, found that by putting twice the number in the smaller coop from the larger, he had the same number in both coops.

8. Two horses trot in the same direction round a circular course 1½ miles long. One goes at the rate cf 8 miles an hour, the other at the rate of 12: how long after starting will they be together again?

9. From a pile of wood 20 feet long, 4 feet wide and 6 feet high, was sold $13.50 worth, at $6 a cord: how much was what remained worth, at $5½ a cord?

10. The head of a fish is 7 inches long; the tail is as long as the head and ⅓ of the body, and the body is as long as the head and tail: how long is the fish?

XII.

1. Eight persons engaged a pleasure boat, but before they start 4 more join them; the expense of each is diminished by $⅔: what did they pay for the boat?

2. If 4 men can remove $\frac{1}{6}$ of a heap of stones in $\frac{6}{12}$ of a day, how many men will be needed to remove the whole heap in $\frac{1}{3}$ of a day?

3. A boy agrees to carry 30 glasses to a certain store for 8 cents a piece, on condition that for each one he broke he should forfeit 12 cents; he received $1.40: How many did he break?

4. A mixture of black and green tea weighing 7 lbs. is worth $3.90; if the proportions of each are interchanged the mixture will be worth $3.80; the black tea is worth 60 cents per lb.: find the price of the green tea.

5. A gardener has an oblong plot of ground $20\frac{5}{6}$ ft. long by $12\frac{1}{2}$ ft. wide, which he wishes divided into square lots of the largest size possible: how many lots will he have?

6. A grocer bought 16 bushels of potatoes; the good at 40 cents a bushel, and the bad at 25 cents. The whole cost $5.35: how many bushels were good?

7. A rope 34 feet in length was broken, so that $\frac{2}{3}$ of the length of the longer piece was equal to $\frac{3}{4}$ of the length of the shorter: what was the length of each piece?

8. A cat is 40 leaps ahead of a dog, and takes 7 leaps to the dog's 4, but 3 of the dog's leaps are equal to 6 of the cat's: how many leaps will the cat take before being caught?

9. Mary meeting some beggars gave each 6 cents, and had 25 cents remaining; had she given each 8 cents, she would have had 3 cents left: how many beggars were there?

10. A garrison of 150 men have provisions for 80 days allowing each man 4 lbs. a day; after 50 days 50 more join them and their daily rations is diminished 1 lb. a day: how long will their supplies last?

XIII.

1. If $2\frac{5}{8}$ yards of cloth cost \$$4\frac{1}{5}$, what will $5\frac{1}{3}$ yards cost?

2. Five men agree to do a piece of work, but two of the men not coming, the work was prolonged $3\frac{1}{2}$ days: in what time could the 5 men do it?

3. A merchant sells 35 inches for a yard and at an advance of $\frac{1}{3}$ on cost: find his gain per cent?

4. What is the time, provided $\frac{2}{3}$ of the time past midnight, plus $1\frac{1}{15}$ hours, equals $\frac{2}{5}$ of the time to midnight?

5. How far can a boy ride with his father who leaves Brampton for Toronto at 7 o'clock in the morning, and can drive the distance, 22 miles, in $3\frac{1}{7}$ hours, so that he may be back at 9 o'clock for school, if he walk at the rate of $3\frac{1}{2}$ miles an hour?

6. A pole is fixed in the bottom of a river. Three feet are in the air, the part in the water is 3 times as long as that in the mud, and the part in the mud is $\frac{4}{15}$ of the rest of the pole: how long is the pole?

7. How many stones 3 feet long by 2 feet wide and $1\frac{1}{2}$ ft. thick will it take to build the walls of a house 18 feet long, 12 feet wide and 3 feet thick, and 4 feet high?

8. A boy engaged with his master for \$80 a year and a suit of clothes, but leaving at the end of 5 months, he received \$24 and the suit of clothes: find its value.

9. A room is 20 feet long and 16 feet wide: what must be the height in order that the area of the floor and ceiling together may be equal to the area of the walls?

10. Two trains, each 88 yards long, when moving in the same direction pass each other in 18 seconds, and in 6 seconds when moving in opposite directions: find the rates of the trains.

XIV.

1. The difference between the interest and the discount on a sum of money for one year and 9 months at 8 per cent. per annum, was $9.80: find the sum of money.

2. A room whose length is 1$\frac{1}{6}$ times its breadth, and height 12 ft., takes 156 yards of paper, 24 inches wide, to cover its walls: what will it cost to carpet the floor with carpet 27 inches wide and $1.25 per yard?

3. The interest on a sum of money for 5 years is $140, and the discount for the same time and rate is $100: find the sum and rate per cent.

Sol.—140—100 = 40 = int. on 100 ∴ $\frac{40}{100}$ = $\frac{2}{5}$ *i. e.*, int. is $\frac{2}{5}$ of principal ∴ disct. = $\frac{2}{7}$; then 140 is $\frac{2}{5}$ of $3502 principal, &c.

4. A grocer bought green tea and black in the ratio of 2 : 1, the former costing 70 cents per pound, the latter 80 cents—the whole costing $44: how much will he make by selling the whole at a uniform price of 90 cents per pound?

5. What amount of accounts must an agent collect in order to pay over $1,100 after retaining 8$\frac{1}{3}$ per cent. for collecting?

6. An agent sold goods on a commission of $2\frac{1}{2}$ per cent., and reserving his commission for both transactions, purchased bank stock on a commission of 2 per cent. on the price paid; his entire commission being $150, find what the goods sold for, and what was paid out for the stock.

7. Find the cost of 3,600 yards of ribon at 2*s.* 11*d.*, and the square of 711.

8. A merchant buys cotton at 12 cents a yard: at what rate per cent. of profit must he sell it so that the money he receives for 50 yards may equal his gain on $24 of outlay?

> *Sol.*—For $24, are bought 200 yds. ∴ gain on 200 yds.=*selling price* of 50 yds.=*cost* of 50 yds+gain on do. ∴ gain on 150 yds.= cost of 50 yds., or $33\frac{1}{3}$ per cent.

9. Coffee at 30 cts., and chicory at 10 cts. a pound are mixed in such proportions that the mixture sold at 30 cents, making a profit of $33\frac{1}{3}$ per cent.: in what proportion were they mixed?

10. A debt after a deduction of 3 per cent. becomes $194: what would it have been after a deduction of 4 per cent?

XV.

1. A grocer by selling 8 lbs. of tea at a certain rate, gained 25 per cent. afterwards he increased his price, giving only 7 lbs. for the same money: how much per cent did he make at his increased price?

2. A merchant marked cloth 25 per cent. in advance of cost, and in selling it used a yard stick 1 inch too short; his entire gain was $12: find the cost price of the cloth?

Sol.—25 per cent. $=\frac{1}{4}$ honest gain; 1 inch gained on 35 $=\frac{1}{35}$, on which there was a gain of 25 per cent. $=\frac{1}{4}$, or $\frac{1}{35}+\frac{1}{4}$ of $\frac{1}{35}=\frac{1}{28}$ $\therefore$ entire gain $=\frac{1}{4}+\frac{1}{28}=\frac{2}{7}$ of cost $=\$42$; $\frac{1}{7}=\$21$, &c. ?

3. A person invests a sum of money in railway stock at 80, and paying 4 per cent. dividends, and $1\frac{1}{2}$ times as much in bank stock at 120, and paying 8 per cent. dividends; his income from both investments is $300; find the amount invested in each kind of stock?

4. I had money at interest at 8 per cent.; afterwards the rate was reduced to 5 per cent., and my annual interest was $168 less: how much had I at interest?

5. A merchant bought 400 lbs. tea and 1600 lbs. sugar, the cost of the latter per lb. being $\frac{1}{6}$ that of the former; he sold the tea at a profit of $33\frac{1}{3}$ per cent., and the sugar at a loss of 20 per cent; his net gain was $60: find his buying prices?

Sol.—Take cost of tea as unit, then tea being 1, sugar $=\frac{1}{6}$ of the same quantity; but there is 4 *times* as much, $\therefore$ tea 1, sugar $\frac{4}{6}=\frac{2}{3}$, gain on former, $\frac{1}{3}$ loss on latter, $\frac{1}{5}$ of $\frac{2}{3}=\frac{2}{15}$ $\therefore$ net gain $=\frac{1}{3}-\frac{2}{15}=\frac{1}{5}=\$60$, and whole $=\$300$ $\therefore$ &c.

6. Write down the result of $39211^2-39101^2$?

7. If the cost of goods had been 20 per cent. more, the gain would have been 25 per cent. less: what was the gain per cent?

8. How much alloy added to 6 oz. of gold 20 carats fine, will make it 15 carats fine?

9. The interest on a principal for 6 years was $261, and the discount $180: find the principal and the rate.

10. Reckoning bank discount at 5 per cent. a person would receive $21 less than the nominal value of a note which has a year to run: what would he receive for the note if *true* discount were deducted?

XVI.

1. Find the sum of the squares and the cubes of the numbers 1, 2, 3, 4, 5, 6.

2. A man is engaged to work at $1 a day and his board, with the understanding that he is to pay 37½ cts. a day for his board when idle: what must be the proportion of working time so that he may just keep out of debt, supposing that he works whole days and not parts in every case?

Sol.—Wages, $$\frac{8}{8}$; loss on idle days=$$\frac{11}{8}$, L. C. M.=$88=11 working or 8 idle days ∴ $\frac{11}{19}$.

3. When 1 guinea is the amount of £1, for what sum is 1 guinea the interest?

Sol.—Amount=$\frac{21}{20}$ ∴ interest=$\frac{1}{20}$ ∴ principal=20 interest=£21.

4. Which is the more profitable, a cord of wood which costs $4.75, and $1.25 for cutting, etc., generating a temperature of 72° in my room for 24 days, or a ton of coal which costs $5, which would generate a temperature of 70° for 25 days, but for a loss of 80 lbs. per ton from dust and dross?

Sol.—80 lbs per ton=4 per cent.=loss=1 day in 25, leaves 24 days same as wood; 72° costs $6 with wood ∴ 1° costs $$\frac{1}{12}$; with coal, 70° costs $5, or 1° costs $$\frac{1}{14}$ ∴ coal better.

5. Sold a horse for $246, and lost 18 per cent. of what he cost: what should be the selling price so that the gain might be 18 per cent. on the cost?

Sol.—18 per cent. cost=\$246, 1 per cent.=\$3, cost=\$300, 18 per cent.=\$54, price=\$354.

6. Bought bananas at 4 for a quarter, and as many at 6 for a quarter; sold them all at 5 for a quarter and lost \$1 on the business: how many bananas were bought, and what did they cost?

7. A dishonest baker set out with a basket of loaves which weighed 4 lbs. instead of 5 lbs. each; he had sold $\frac{3}{4}$ of his lot, by which his fraudulent gain was 48 cts., when the rest were confiscated, and his total loss was equal to $\frac{3}{5}$ of his loaves: find the number of loaves.

Sol.—Gain=$\frac{1}{5}$ price, gain on $\frac{3}{4}$=48 cts., on whole 64 cts. ∴ 64 × 5=\$3.20, real value of bread; $\frac{1}{4}$ confiscated worth $\frac{4}{5}$ of $\frac{1}{4}$=$\frac{1}{5}$ real value of bread=64 cts. partial loss ∴ total loss=12 cts. ∴ price=16 cts. ∴ 20 loaves.

8. A boy bends his hoop into the form of a triangle whose sides are 7, 17, and 20 inches: what was the diameter of the hoop, given circumference=$\frac{22}{7}$ diameter?

9. If a square rood be divided into 10 equal squares, what is the length of a diagonal of each of these smaller squares?

Sol.—Square rood=1210 yards ∴ smaller square=121 square yards ∴ 1 side=11 yds., and diagonal=$\sqrt{2 \times 11^2}$ or $11\sqrt{2}$ yds.

10. The nickel cent is 1 inch in diameter: find the area of the space enclosed by 3 coins, placed so as to touch each other, knowing that the area of a circle is equal to 3·1416 square of radius, and area of equilateral triangle = 433 square of side.

XVII.

1. What number must be added to or subtracted from the product of the sum and difference of 11 and 14, so that it will contain 13 evenly ?

2. How many times greater is the L. C. M. of 4, 6 and 8 than the H. C. F. of $\frac{1}{2}$ and $\frac{2}{3}$?

3. The L. C. M. of two numbers is 63 and their G. C. M. is 3; one of the numbers is 9 : find the other ?

4. A grocer bought 100 geese and turkeys for $65 ; for the geese he paid 30 cts each and for the turkeys 80 each : find the number of each kind.

5. A father dying leaves his estate to his two sons, ages 19 and 20 respectively, to be divided so that each shall have equal amounts at maturity. In what proportion should it be divided, interest at 5 per cent per annum ? *(a)* Simple Interest. *(b)* Compound Interest.

6. A man wishing to sell a horse asks 20 per cent. more than it cost ; he finally sold it for 25 per cent. less than his asking price and lost $15 on cost : what was his asking price ?

7. What should be the Policy of Insurance at 5 per cent. on a diamond necklace worth $380, so that if lost the owner shall recover the premium and value of the necklace ?

8. If the true discount off a note of $420 for a certain time be \$20, what will be the discount off a note of \$100 for three times as long?

9. Two qualities of vinegar, No. 1 and No. 2, are mixed in the proportion of 2 : 3. No. 1 is worth 15 cents a gallon, and No. 2 10 cents a gallon: what should be the marked price per gal. so that there may be a gain of 25 per cent. on cost after lowering the asking price 10 per cent?

10. Find the perimeter of a rectangular plot of land whose length is $2\frac{1}{2}$ times its breadth and contains 1,000 sq. perches

11. A speculator sells his goods at a profit of 50 per cent., but the purchaser fails and pays only 50 cts. on the \$1: how much per cent. did the speculator lose or gain on his venture?

XVIII.

1. Two partners gain \$480; A puts in \$560 more than $\frac{1}{3}$ of the stock, and his share of the gain is \$260: find the stock contributed by each.

2. What is the length in yards of a piece of cloth originally $1\frac{1}{3}$ yards wide, provided the cloth after shrinking 10 per cent, in length and width, contains 9 full square yards?

3. Paid \$34.50 for corn at \$0.75, wheat at \$1, and oats at \$0.50 a bushel; sold $\frac{2}{3}$ of the corn and $\frac{1}{2}$ of the wheat at 50 per cent. advance, gaining on the corn $\frac{2}{3}$ as much as on the wheat, and on the sale, the cost of the oats: how much of each did I buy?

4. If an article had cost me 10 per cent. less, my gain would have been 11 per cent more: what was my rate of gain or loss?

5. At what time between 5 and 6 o'clock do the hour and minute hands make equal acute angles with the line from 12 to 6 ?

6. At a certain time between 8 and 9 o'clock the minute hand was between 9 and 10, within an hour afterwards the hands had changed places : what was the time first mentioned ?

7. A boy sold two knives at the same price ; on the one he gained 20 per cent. on the other he lost 20. He lost two cents by the bargain : find selling and cost price of each ?

8. A sold a sheep, and lost 25 per cent ; if he had paid $1 more for it he would have lost 40 per cent : what did he pay for the sheep ?

9. Bought a hind and a fore-quarter of lamb, weighing together 24 lbs. ; the hind-quarter was 12 cts. and the fore-quarter 8 cts. per lb. ; the two cost me 8 cts. more than if I had bought the whole at an average price of 10 cts. per lb. : how much did each quarter weigh ?

10. The true discount on a sum of money for a year and a half at 8 per cent. being $12, find the true discount on the same sum and for same time at $5\frac{5}{39}$ per ce

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